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ELECTRICAL TECHNIQUES FOR COMPENSATION OF THERMAL TIME LAG OF THERMOCOUPLES AND RESISTANCE THERMOMETER ELEMENTS

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SUMMARY

Basic electrical networks are described that compensate for the thermal time lag of thermocouple and resistance thermometer elements used in combustion research and in the control of jet power plants. The measurement or the detection of rapid temperature changes by use of such elements can thereby be improved.

For a given set of operating conditions, networks requiring no amplifiers can provide a thirtyfold reduction in effective time lag. This improvement is obtained without attenuation of the voltage signal but results in a large reduction in the amount of electric power available because of an increase in the output impedance of the network. Networks using commercially available amplifiers can provide a thousandfold reduction in the effective time lag without attenuation of the alternating voltage signal or of the available electric power, but the improvement is often obtained at the expense of loss of the zero-frequency signal. The completeness of compensation is limited by the extent of off-design operation required.

INTRODUCTION

The control of jet engines and the fundamental study of the dynamic combustion phenomena associated with such power plants involve the measurement of fluctuating gas temperatures. When a thermocouple or resistance thermometer is used as the primary element for such temperature measurements, the thermal lag of the element, caused by slow rate of heat transfer between the element and the surrounding gas, limits the ability of the element to follow such fluctuations. The thermal lag is such that, as the frequency of the fluctuations increases, the amplitude of the fluctuations indicated by the element decreases and also lags in time behind the true temperature. Sometimes it is possible to use a primary element of sufficiently small size to achieve adequately rapid response. In many cases, however, such small elements have insufficient mechanical strength and inadequate service life, so that larger elements with appreciable lag must be employed.

Since the output of the primary element is an electric voltage, electrical networks can be used to compensate for the time lag and the reduction in response amplitude and thereby to permit measurement of rapidly changing temperatures with improved fidelity. Compensation of this type, generally involving specially designed amplifiers, has been the basis of hot-wire anemometry (for example, references 1 to 3). Such compensation is also provided by the equalizing networks used in communication engineering.

Some basic networks that can be used individually or in combinations to compensate for the thermal lag of the primary electric thermometer elements used in jet engine operation are described herein. In those combinations which include amplifiers, the amplifiers are of a semi-industrial type, available commercially, and capable of extended use without adjustment or service.

In general, the compensating network is designed to correct for a given amount of primary-element thermal lag which can be described by a characteristic "time constant" of the element. The magnitude of this time constant is a function of the mechanical design of the element and of the aerodynamic conditions in which the element is used (reference 4). Since the compensation employed is generally correct for only one set of aerodynamic conditions, the effects upon the performance of the compensated system of deviations from the design point must be considered. If the time constant of the primary element has been determined experimentally at one set of aerodynamic conditions, it is generally possible to compute the time constant at other sets of aerodynamic conditions and then to adjust the electrical compensating system for the new set of conditions.

The nature of the response of the basic primary element and the principles of electrical lag compensation will first be reviewed herein. Then, after introduction of the operational terminology of the "transfer function," several basic types of compensator circuit, both alone and in combination with amplifiers, will be examined in detail. The next section will consider the effects of off-design operation on performance. Finally, the performance of two specific assemblies of apparatus will be described and their particular fields of usefulness indicated.

In the presentation of compensating circuits, two basic forms of compensator, the resistance-capacitance (RC) and the resistance-inductance (RL) types, will first be described, together with one simplification of the RC type, the differentiating type. These compensators suffer from

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a deficiency in that the improvement in effective time constant (and therefore in frequency response) is achieved only at the expense of a corresponding attenuation of signal. The addition of an amplifier can correct for this attenuation. The amplifier introduces noise that tends to mask the signal; however, the addition of a transformer as a preamplifier permits attainment of a more favorable signal-to-noise ratio. A feedback amplifier can be designed to combine the functions of amplifier and compensator. Finally, there will be described another basic type of compensating network, the transformer type, that combines the functions of noise-free amplifier and of compensator.

The analysis and results presented herein constitute a portion of a temperature measurements research program being conducted at the NACA Lewis laboratory.

TIME LAG COMPENSATOR NETWORKS

Response of Primary Element

Differential equation of primary-element response. - If a thermocouple or a resistance thermometer element is subjected to a changing gas temperature $T_0(t)$, the time rate of change dT_1/dt of the element temperature $T_1(t)$ is proportional to the temperature difference $(T_0 - T_1)$. This may be expressed (reference 4) as

$$\tau_1 \, (dT_1/dt) = T_0 - T_1 \quad \text{or} \quad \tau_1 \, (dT_1/dt) + T_1 = T_0 \tag{1}$$

where the factor of proportionality τ_1, the "time constant," is characteristic of the element, of the gas properties, and of the gas flow conditions. The value of the time constant completely defines the response of the element temperature to a changing gas temperature. (All symbols used herein are defined in appendix A.)

Response to two simple types of gas temperature change. - The response characteristics of the primary element are visualized most simply by reference to figure 1, which describes the response to two types of input change:

(a) The curve of figure 1(a) shows that in response to a sudden "step change" ΔT_0 in gas temperature, the approach of the element temperature to its new value is along an exponential curve and covers 63 percent, 86 percent, 95 percent, and 98 percent of the interval ΔT_0 in time τ_1, $2\tau_1$, $3\tau_1$, and $4\tau_1$, respectively.

(b) In response to a steady sinusoidal variation in gas temperature of amplitude $\overline{T}_0$ and angular frequency ω, the temperature of the primary element will oscillate with reduced amplitude $\overline{T}_1$ and will lag in phase and time, as shown in figure 1(b).

The value of the amplitude ratio $\overline{T}_1/\overline{T}_0$ is shown in figure 1(c) as a function of the angular frequency ω. When $\omega = 1/\tau_1$, the amplitude ratio has dropped to $1/\sqrt{2}$. It is customary to term the response "flat" to the point where $\omega = 1/\tau_1$ and to term this point the "limit" of response, or the "cut-off point." 2388

Curves of time lag and phase lag are shown in figure 1(d) as a function of angular frequency. When $\omega = 1/\tau_1$, the time lag is $\pi\tau_1/4$ and the phase lag is 45°. The time lag is substantially equal to τ_1 for $\omega\tau_1 < 1$.

<u>Response in terms of voltage</u>. - The output emf e_1 from a thermocouple or from a resistance-thermometer bridge can be represented as being proportional to the difference between the temperature T_1 of the primary element and some reference temperature T_R such that

$$e_1 = Q(T_1 - T_R) \tag{2}$$

For a thermocouple, T_R is the "cold-junction" temperature; for a resistance thermometer, T_R is the "reference resistor" temperature. In the case of the thermocouple, Q is the thermoelectric power; in the case of the resistance thermometer, Q is the product of the temperature coefficient of resistance of the thermometer element, the voltage applied to the bridge, and a dimensionless function of the bridge circuit resistances.

A fictitious emf e_0 will be defined as the equivalent emf that would be produced by a primary element at a temperature T_0

$$e_0 = Q(T_0 - T_R) \tag{3}$$

Substituting equations (2) and (3) into equation (1) yields

$$\tau_1 \,(de_1/dt) + e_1 = e_0 \tag{4}$$

Principles of Electrical Lag Compensation

Electrical lag compensation could be produced if an electrical network were found that could perform upon the primary element output signal e_1 the operations described by the left side of equation (4). The result so obtained would be the voltage e_0 representative of the true gas temperature.

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A network that approximates the desired operations is shown in figure 2(a). Suitable choice of the elements R and C can produce a current through R proportional to the time derivative of input voltage; suitable choice of R_C can produce an additional direct current through R proportional to the input voltage; by proper choice of the constants of proportionality, the total current then flowing through R, and therefore the voltage across R, can be made approximately proportional to e_0, as given by equation (4).

A second network that approximates the desired operation is shown in figure 2(b). Suitable choice of the elements R, R_L, and L can produce a voltage across L approximately proportional to the time derivative of input voltage and a voltage across R_L proportional to the input voltage; by proper choice of the constants of proportionality, the sum of the voltages across L and R_L can be made approximately proportional to e_0, as given by equation (4).

A third network that approximates the desired operations is shown in figure 2(c). Suitable choice of the elements R_p, L_1, and N can produce a primary current approximately proportional to input voltage and a secondary emf proportional to the time derivative of primary current and therefore to the time derivative of input voltage. With suitable choice of circuit constants, the autotransformer connection can then produce the summation required by equation (4).

In order to establish a mathematical basis for such compensating methods and for more complex refinements of the basic principles and to define the limits of approximation, it is convenient to introduce the concept of the transfer function.

Operational Terminology and the Transfer Function

Operational terminology. - The response of the simple system represented by equation (1) to an applied temperature $T_0(t)$ of arbitrary form is given by

$$T_1 = c\epsilon^{-t/\tau_1} + \epsilon^{-t/\tau_1}\int \epsilon^{t/\tau_1} (T_0/\tau_1)dt \tag{5}$$

where c is a constant determined by the initial value of T_1. However, it is more convenient for purposes of discussion to present the solution in a symbolic form wherein the symbol p replaces the differential operator $d(\ )/dt$ of equation (1). Then the solution of equations (1) and (4) can be indicated by 2388

$$T_1 = \frac{1}{1 + \tau_1 p} T_0 \tag{6a}$$

$$e_1 = \frac{1}{1 + \tau_1 p} e_0 \tag{6b}$$

It can be shown that, as long as p is multiplied only by constants, it may be treated as an algebraic quantity where p represents $d(\ )/dt$, p^2 represents $d^2(\ )/dt^2$, $1/p$ represents $\int(\ )dt$, and so forth (reference 5). The usefulness of this method of representation is revealed by noting that (1) linear differential equations of more complex form than equation (1) can be solved by the operational methods of Heaviside or Laplace, where the appropriate operator replaces p; and (2) that when e_0 is represented as a Fourier series

$$e_0 = Q(T - T_0) = \sum_{n=0}^{\infty} \bar{B}_n \sin (n\omega t - \theta_n) \tag{7}$$

the solution of such differential equations is obtained in complex representation by formally replacing p by $j\omega$ (references 5 to 7). The amplitude ratio, the time lag, and the phase lag, at each angular frequency ω, are then readily derivable. For the primary element with characteristic given by equation (6) and input e_0 given by equation (7), the response e_1 is

$$e_1 = Q(T_1 - T_0) = \sum_{n=0}^{\infty} \frac{\bar{B}_n}{\sqrt{1 + n^2\omega^2\tau_1^2}} \sin (n\omega t - \theta_n - \varphi_n) \tag{8a}$$

where

$$\varphi_n = \tan^{-1} (n\omega\tau_1) \tag{8b}$$

In order to take advantage of the convenience afforded by the operational terminology, all systems treated in this report will be assumed linear.

Definition of transfer function. - The dimensionless ratio of the output quantity (expressed as a function of p) divided by the input quantity (also expressed as a function of p) is defined as the transfer function $Y(p)$ of a system obeying a linear differential equation. From equation (6) the transfer function of the primary element is

$$Y_1(p) = \frac{1}{1+\tau_1 p}. \tag{9}$$

The advantages of expressing the characteristics of systems such as electrical networks by means of transfer functions are

(a) The equivalent transfer function, and hence the response characteristics, of a system composed of several elements can be obtained from the transfer functions of the individual elements by simple algebraic methods.

(b) The response of a system having a transfer function $Y(p)$ to an impressed sinusoidal voltage $\bar{E} \sin(\omega t-\varphi)$ is $[\bar{E} \sin(\omega t-\varphi)]\, Y(j\omega)$, where $j\omega$ has been substituted for the operator p.

Transfer function of cascaded system. - When several elements are joined in cascade as in figure 3(a), that is, when the output of one element is fed into the input of the adjacent element, the resultant transfer function is the product of the transfer functions of the individual elements. If this product is a constant, one transfer function is said to be the inverse of the others.

As an example, if the primary element output e_1 given by equation (6b) is applied to an electric network whose transfer function is

$$Y_4(p) = (1/A_{dc})(1+\tau_1 p) \tag{10}$$

where A_{dc} is a constant (representing the attenuation at zero frequency), the resultant product $Y_1(p)\ Y_4(p)$ will be equal to $1/A_{dc}$, and the output e_4 will be proportional to the fictitious emf e_0 and hence to the applied temperature T_0. Therefore, at all frequencies at which changes in T_0 may occur,

$$e_4 = (1/A_{dc})e_0 \tag{11}$$

Networks that have transfer functions inverse to the primary element over a limited frequency range will be described under Compensating Circuits. These networks, in their working region, have the transfer function of equation (10).

Transfer function of additive system. - If the outputs of several networks which have a common input are added as shown in figure 3(b), the resultant transfer function is equal to the sum of the transfer functions of the individual networks.

Combination of cascaded and additive systems. - As an example, the arrangement of networks shown in figure 3(c), with the transfer functions of the various networks as shown in the figure, yields an output emf e_2 equal to the input emf e_0.

Compensating Circuits

Terminology. - In following paragraphs, various compensating systems will be treated. These systems will generally consist of basic components conveniently designated transformers, amplifiers, compensators, and so on. The terminology is clarified by the block diagram of figure 4 which shows the most common combination of components and the subscripts assigned to the components, to the potential differences at their terminals, and to the constants by which they are characterized. In this figure, the term "compensator" has been assigned to that component the chief function of which is to correct for the reduction in response amplitude of the primary element with increasing frequency of the applied temperature variation. Unless specifically indicated otherwise, the terminology of figure 4 will apply to material presented in subsequent paragraphs of this discussion of compensating circuits.

A compensating system (including one consisting of a single component) will be characterized chiefly by two parameters: The frequency-response improvement factor F is equal to the ratio of the upper cut-off frequency of the compensated system to the upper cut-off frequency of the uncompensated primary element; the over-all gain factor G is equal to the ratio of the voltage output of the system (as it appears at the detector input terminals) in the region of flat frequency response to the equivalent electric input signal e_0 to the primary element.

RC and RL compensating circuits without amplifier. - Two basic networks which approximate the characteristic defined by equation (10) are shown in figures 2(a) and 2(b). The transfer function of these compensators in the idealized case of negligible source impedance and load admittance is

$$Y_4(p) = \left(\frac{1}{r}\right)\frac{1+\tau_4 p}{1+(\tau_4/r)p} \tag{12a}$$

where τ_4 and r are defined in figure 2. The value of $|Y_4(j\omega)|$ is shown in figure 2(d). For the condition $\tau_4 = \tau_1$, where the compensator time constant τ_4 is equal to the primary element time constant τ_1, the product $Y_4(p)\ Y_1(p)$, which describes the performance of a system consisting of the primary element plus compensator in cascade, is

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$$Y_4(p)\ Y_1(p) = \left(\frac{1}{r}\right)\frac{1}{1+(\tau_4/r)p} \tag{13}$$

This equation represents a linear first-order system, with the effective time constant reduced by a factor of r from the value of τ_1 for the primary element alone. For this idealized system, $F = r$ and $G \doteq 1/r$. The system has the following additional characteristics:

(a) The response to a steady sinusoidal input $\overline{E}_0 \sin \omega t$ is a sine wave of amplitude $\overline{E}_0 \Big/ \left[r\sqrt{1+\omega^2\ (\tau_4/r)^2}\right]$ and phase lag $\tan^{-1}\ (\omega\tau_4/r)$. This response is shown in figures 5(a) and 5(b). The range of "flat" response is extended by a factor $F = r$ over that of a simple primary element. The attenuation of a zero frequency signal is $A_{dc} = 1/G = r$.

(b) The amount r by which the frequency response can be improved is limited by the amount of amplitude attenuation that can be tolerated.

(c) The response to an impressed step change of magnitude Δe_0 is described by an exponential curve having a time constant τ_1/r and a magnitude of change $\Delta e_0/r$ (fig. 5(c)). Consequently, the improvement in time constant is accomplished at the expense of a corresponding reduction in amplitude Δe_4 of the output signal.

(d) The maximum value of time constant which can be compensated is determined by the components of the compensator. For the compensator of figure 2(a), the RC network, the maximum value of $R_C C$ offers no practical limitation in the applications generally encountered. In the case of the RL network of figure 2(b), the maximum possible value $L/R_{L,min}$ of the time constant (where $R_{L,min}$ is the resistance of the inductor) is usually a function of the weight of the inductor. For commercially available inductors, a value of $L/R_{L,min}$ of 0.05 second is typical for a weight of 1 pound and the time constant increases approximately as the square root of the weight of the inductor.

(e) The output impedance of the network varies with frequency. For the RC network of figure 2(a), the output impedance has a maximum value of approximately R at zero frequency and approaches the resistance of the primary element at very high frequencies. The output impedance of the RL network of figure 2(b) has a minimum value of approximately R_L at zero frequency and approaches R at very high frequencies.

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Effect of loading on RC and RL compensators. - The transfer function given in equation (12) is derived for the conditions of a source of negligible impedance and a load of negligible admittance. If a compensator of the type of figure 2(a) or of figure 2(b) is connected to a source of resistance R_σ and to a load of resistance R_λ (fig. 6), the effect is to change the form of the transfer function. Although the value of τ_4 is unchanged in all cases, the factor by which the d-c. signal is attenuated is no longer equal to r, nor does r any longer represent the factor by which the frequency response is extended when the compensator is connected to a primary element of time constant $\tau_1 = \tau_4$. Under the condition of nonzero source resistance and finite load resistance, the gain factor of the combination of compensator and primary element will be denoted G_4 and the factor by which the frequency response is extended will be denoted F_4. The transfer function is

$$Y_4(p) = G_4 \frac{1+\tau_4 p}{1+(\tau_4/F_4)p} \qquad (12b)$$

as compared with equation (12a) for the idealized case. The changes in the quantities $(1/G_4)$ and F_4 are given in table I. The effect of adding a source resistance R_σ and a load resistance R_λ may be summarized as follows:

(a) For the RC compensator of figure 6(a), the effect of keeping $R_\sigma = 0$ and making R_λ finite is to decrease G_4 and to increase F_4 in the same proportion. The effect of keeping R_λ infinite and making $R_\sigma > 0$ is to decrease both G_4 and F_4.

(b) For the RL compensator of figure 6(b), the effect of keeping $R_\sigma = 0$ and making R_λ finite is to decrease both G_4 and F_4. The effect of keeping R_λ infinite and making $R_\sigma > 0$ is to decrease G_4 and to increase F_4 in the same proportion.

(c) The combined effect for both the RC and the RL compensators of making $R_\sigma > 0$ and R_λ finite is to decrease both G_4 and the product F_4G_4, so that the attenuation is increased without a proportionate increase in frequency response improvement factor. The relations between G_4 and F_4 are, for the RC compensator,

$$F_4 = \frac{1 + \rho_C}{G_4 + \rho_C} \tag{14}$$

where

$$\rho_C = \frac{R_\sigma}{R_C}$$

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For the RL compensator,

$$F_4 = \frac{1 + \rho_L}{G_4 + \rho_L} \tag{15}$$

where

$$\rho_L = \frac{R_L}{R_\lambda}$$

<u>RC differentiating circuit used as compensator</u>. - A simplified case of the RC compensator of figure 2(a) is obtained when the capacitor shunt resistor R_C is infinite. The resulting transfer function is

$$Y_4(p) = \frac{\tau_4'p}{1 + \tau_4'p} \tag{16}$$

where $\tau_4' = RC$. This network is usually known as a differentiating circuit since the differentiating function is performed for the range of frequencies in which $\omega\tau_4' < 1$. This network is also effective in compensating for the response of a primary element in the high-frequency region where $\omega\tau_1 > 1$, provided that τ_4' is chosen so that $\omega\tau_4' < 1$. It will be noted that τ_4' is no longer chosen equal to τ_1. The increase in the upper frequency limit is $F = \tau_1/\tau_4'$ and this factor also describes the reduction $1/G$ in signal amplitude for the operating range. The differentiating circuit will not pass the average value of the signal and therefore is useful only for measuring the alternating components of the gas temperature in the frequency range

$$\left(\frac{1}{\tau_1}\right) < \omega < \left(\frac{1}{\tau_4'}\right)$$

<u>RC and RL compensating circuits with amplifiers</u>. - The compensating circuits just described are defective in that, although the effective time constant is reduced by a factor F (and the frequency range is correspondingly extended by the same factor), the amplitude of any

applied signal is attenuated by a factor $1/G$ that is equal to or greater than F. This defect can be remedied by the addition of an amplifier. If an amplifier the transfer function of which is $Y_3(p)$ is added to the circuit after the primary element, as in figure 7, the resultant transfer function of the system will be equal to the product $Y_4(p)\ Y_3(p)\ Y_1(p)$. The response of the system is described for several possible conditions:

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(a) If the amplifier is a distortionless d-c. amplifier (that is, one which has a flat frequency response and linear phase shift from zero frequency up to the maximum frequency where compensation is to be achieved), the effects of the amplifier are an increase in the signal voltage equal to the voltage amplification factor μ_3 of the amplifier, a constant shift in time, and the introduction of a certain noise voltage. The magnitude of the time shift is negligible in most practical applications involving temperature measurement; the problem of noise is discussed in the section Amplifier noise. The transfer function of the amplifier-compensator combination is

$$Y_4(p)\ Y_3(p) = \mu_3 G_4 \frac{1 + \tau_4 p}{1+(\tau_4/F_4)p} \tag{17}$$

For this system, $F = F_4$ and $G = \mu_3 G_4$.

(b) If the amplifier has a high-frequency cut-off point (the point at which the amplification factor is reduced by a factor $1/\sqrt{2}$) at $\omega = \omega_{3,b}$ such that

$$Y_3(p) = \frac{\mu_3}{1+p/\omega_{3,b}} \tag{18}$$

where μ_3 is the voltage gain in the region of flat response, then for the condition $\tau_4 = \tau_1$, the frequency response of the system corresponding to the transfer function $Y_4(p)\ Y_3(p)\ Y_1(p)$ has the appearance shown in figures 8(a) and 8(b). When $\omega_{3,b} = F_4/\tau_4$, the high-frequency cut-off point is reduced to 64 percent of the value for $\omega_{3,b} = \infty$. The response of the system to a step change is shown in figure 8(c). The effective time constant, which will be defined as the time required for the output to reach 63 percent of the total change for a step input, is approximately equal to $\tau_4/F_4 + 1/\omega_{3,b}$. Correspondingly, the approximate value of F is $\left(\frac{1}{F} + \frac{1}{\tau_4 \omega_{3,b}}\right)^{-1}$ and the value of G remains $\mu_3 G_4$.

(c) If the amplifier has a low-frequency cut-off point at $\omega_{3,a}$ (the point at which the amplification factor is reduced by a factor $1/\sqrt{2}$) such that

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$$Y_3(p) = \frac{\mu_3 p/\omega_{3,a}}{1+p/\omega_{3,a}} \tag{19}$$

where μ_3 is the voltage gain in the region of flat response, then for the condition $\tau_4 = \tau_1$ the frequency response of the system, corresponding to the transfer function $Y_4(p)\ Y_3(p)\ Y_1(p)$, has the appearance of figures 9(a) and 9(b). For a step change, since the system does not pass the average value of the signal, the output ultimately drops to zero, as shown in figure 9(c). Figure 9(d) describes the maximum value attained in response to a step change and the time to reach this maximum value for various values of $\omega_{3,a}$. For this system, $F = F_4$ and $G = \mu_3 G_4$.

(d) If the amplifier has both low- and high-frequency cut-off, its transfer function is

$$Y_3(p) = \frac{\mu_3 p/\omega_{3,a}}{1+p/\omega_{3,a}} \frac{1}{1+p/\omega_{3,b}} \tag{20}$$

and its frequency response is a composite of the responses shown in figures 8(a), 8(b), 9(a), and 9(b).

In the circuit shown in figure 7, the signal applied to the amplifier varies widely in amplitude as the frequency changes, and an important requirement imposed on any of the simple amplifiers considered is that the amplification factor remain constant over the range of amplifier input voltages delivered by the primary element throughout the usable frequency band. Thus, if compensation is to be obtained up to a frequency that is 1000 times the cut-off frequency of the uncompensated primary element, the amplifier must provide constant gain μ_3 over a 1000 to 1 input voltage range. If it is found that the amplifier cannot operate over the required range of input voltage, the positions of the compensator and the amplifier can be interchanged as shown in figure 10. This arrangement results in a constant input voltage amplitude at a much lower level. However, such an alternate arrangement often aggravates the problems of noise and of impedance matching.

It is often desirable to utilize a-c. amplifiers rather than d-c. amplifiers in order to gain advantages in freedom from drift. In such cases, response down to zero frequency may be retained by use of one of the techniques described later.

Amplifier noise. - If a vacuum-tube amplifier has its input terminals connected to an external source, which can be represented by a zero-impedance generator in series with an external impedance, and if the external source emf is reduced to zero, a certain random voltage will nevertheless exist at the output terminals of the amplifier. In the temperature measuring circuits considered, this voltage, called the noise voltage, is caused almost entirely by the vacuum tubes of the first stage since the thermal-agitation noise (Johnson noise) of the low impedance primary element is generally of negligible magnitude, and since hum pickup in the leads and components can generally be reduced to negligible proportions by the use of twisted leads and careful electrostatic and magnetic shielding. The tube-noise voltage is made up of components of a very large range of frequencies, is proportional to the square root of the frequency pass band of the amplifier, and is therefore reduced by limiting the pass band.

The noise voltage $E_{3,N}$ which exists at the amplifier output can be referred to the input by dividing by the amplifier gain μ_3. This fictitious voltage $E_{2,N}$ is called the equivalent input noise voltage of the amplifier. The noise also appears at the compensator output as a voltage $E_{4,N}$. For the RC or RL compensator previously considered, $E_{4,N}$ is less than $E_{3,N}$ since the compensator always attenuates the signal, with greater attenuation at the lower frequencies. The actual magnitude of $E_{4,N}$ will therefore depend on the frequency spectrum of the noise, on the response characteristics of the compensator, and on the high-frequency cut-off point of the system. The ratio of the root-mean-square value E_0G of the signal voltage appearing at the output of the compensator to the root-mean-square value $E_{4,N}$ of the noise voltage is called the signal-to-noise ratio σ_4. The smallest allowable signal-to-noise ratio depends on the application. A signal-to-noise ratio of 3 is often adequate.

The amplifier noise voltage, the smallest allowable signal-to-noise ratio, and the amount by which it is desired to extend the frequency response determine the minimum signal $E_{0,min}$ that can be detected, corresponding to a temperature sensibility (smallest detectable signal) $T_{0,min}$, in conformance with the relation between F and G and the relation

$$E_{0,min} = \frac{E_{4,N}\sigma_4}{G} \tag{21a}$$

Since the compensator extension of the frequency range by a factor F is associated with a corresponding attenuation 1/G of the signal from the primary element, the input noise voltage of the amplifier limits the frequency band width possible for a given minimum signal.

For an RC or RL type compensator connected between an amplifier of very low output resistance and a very high resistance load, equation (21a) can be written

$$E_{0,min} = \frac{E_{4,N}\sigma_4 F}{\mu_3} \leq E_{2,N}\sigma_4 F \tag{21b}$$

Transformer used as preamplifier. - The restriction imposed by equation (21) seriously limits the band width that can be achieved by the use of commercially available amplifiers, which have a rather high input noise level. If the application permits, the band width can be extended considerably by the use of a high quality step-up transformer to increase the voltage level of the signal without a corresponding increase in noise (fig. 11). The improvement is generally accomplished at the expense of loss of the d-c. response (unless special techniques described in a later section are employed); the effect is the same as that shown in figure 9 for an amplifier with low-frequency cut-off.

A voltage gain up to 140 is attainable with commercially available transformers. The minimum signal $E_{0,min}$ that can be detected is given by equation (21a), and for the RC or RL type compensator can be written

$$E_{0,min} = \frac{E_{4,N}\sigma_4 F}{\mu_2\mu_3} \leq \frac{E_{2,N}\sigma_4 F}{\mu_2} \tag{21c}$$

where μ_2 is the transformer voltage gain (equal to the ratio of transformer output voltage to primary element output emf e_1) and is equal to the turns ratio N when the transformer is not appreciably loaded by the amplifier and the primary element is of negligible resistance. For illustration, the minimum root-mean-square voltage $E_{0,min}$ and the corresponding temperature sensibility $T_{0,min}$ that are obtained at a signal-to-noise ratio $\sigma_4 = 3$ are given in table II for various values of frequency-response improvement factor F, transformer voltage gain μ_2, and noise voltage $E_{2,N}$.

Certain relations must exist between the impedances of the primary element, the transformer, and the amplifier in order that the improvement represented by equation (21c) shall be appreciable. The impedance of the primary element must be low compared with the input impedance of the amplifier in order to obtain full advantage of the insertion of a transformer having a high turns ratio. The general equation for the gain of a transformer connected to its source and load resistances is derived in appendix B and is given by

$$\mu_2 = \frac{N}{\left[1 + \left(\frac{R_s}{R_\lambda}\right) + N^2\left(\frac{R_1+R_p}{R_\lambda}\right)\right]} \tag{22a}$$

For a given primary element resistance R_1 and a given amplifier input resistance $R_{3,2}$, maximum voltage gain will be achieved when a transformer is selected having rated primary and secondary impedances equal to R_1 and $R_{3,2}$, respectively. The gain will then be given approximately by

$$\mu_{2,max} = \frac{1}{2}\sqrt{R_{3,2}/R_1} = \frac{N}{2} \tag{22b}$$

where N is the transformer turns ratio. Conversely, for any given transformer the voltage gain can be made to approach N if the primary element resistance is made low compared with the rated primary impedance of the transformer and if the amplifier input resistance is made large compared with the rated secondary impedance of the transformer. For the system shown in figure 11, $F = F_4$ and $G = \mu_2\mu_3G_4$.

Techniques for maintaining d-c. response. - When an a-c. amplifier is used, or when a transformer is used as a preamplifier to match the primary element resistance to the amplifier input resistance, the resulting loss of d-c. response may be restored by the use of one of the circuits shown in figure 12. Figure 12(a) shows the combination of a pair of synchronous choppers before and after an a-c. amplifier, with the matching transformer relocated to a position immediately after the input chopper. In this case, the frequency band width may be limited by the chopper frequency used and by any necessary filtering. Figure 12(b) shows how the d-c. emf from a second primary element may be injected in series with the output from the compensator. The circuit of figure 12(b) involves the application of the additive network theorem stated earlier.

Negative feedback amplifier. - It is possible to combine the amplifier and compensator functions by inserting into the amplifier a feedback loop the transfer function $\beta(p)$ of which is of the same form as that of the primary element (fig. 13). Here

$$\beta(p) = \beta_{dc}\frac{1}{1+\tau_4 p} \tag{23}$$

where β_{dc} is the d-c. feedback factor. Negative feedback is obtained if β_{dc} is a negative quantity. The gain of the amplifier with feedback is then given by (reference 8)

$$Y_{34}(p) = \frac{\mu_3}{1-\mu_3\beta(p)} \tag{24}$$

which is equal to

$$Y_{34}(p) = \mu_3 G_{34} \frac{1+\tau_4 p}{1+(\tau_4/F_{34})p} \tag{25}$$

where

$$1/G_{34} = F_{34} = 1 - \beta_{dc}\mu_3$$

Equation (25) is of the same form as equation (17), and the previous comments on response apply.

The input voltage to the amplifier is

$$e_0 G_{34} \frac{1}{1+\tau_1 p} \frac{1+\tau_4 p}{1+(\tau_4/F_{34})p} \tag{26}$$

and therefore, for the condition $\tau_4 = \tau_1$, the input is constant over the frequency range $0 \leq \omega \leq F_{34}/\tau_4$ in which compensation is to be achieved. Consequently, this type of amplifier is not required to operate over a large range of input voltage.

<u>Transformer used as compensator</u>. - The transformer-type compensator also combines the functions of voltage amplifier and compensator. The transfer function of a simple transformer connected to source and load resistances, as shown in figure 14(a), is derived in appendix B on the assumption that capacitance and iron loss effects can be neglected. The transfer function when this transformer is connected as an autotransformer, as shown in figure 14(b), is derived in appendix C.

In the idealized case where the source resistance is negligible compared with the transformer primary circuit resistance and no current is drawn from the transformer secondary, the transfer function of the autotransformer is

$$Y_{34}(p) = \frac{1+\tau_4 p}{1+\left(\frac{\tau_4}{N+1}\right)p} \tag{27}$$

The value of $Y_{34}(j\omega)$, as a function of frequency, is shown in figure 2(e). The transfer function and the response characteristics are of the same form as those of the RC or RL compensators of figures 2(a) and 2(b) (see fig. 5) except that the output voltage is of the same magnitude as the input voltage rather than being attenuated by the frequency response improvement factor. The time constant τ_4 is

$$\tau_4 = \frac{(N+1)L_1}{(R_p+R_p')} \tag{28a}$$

where N, L_1, and R_p are the transformer turns ratio, primary inductance, and primary resistance, respectively. The time constant is reduced by adding external resistance R_p' in series with the transformer primary.

It is to be noted that the transformer is not being used in the region of its flat frequency response, as in customary transformer applications. Instead, the transformer is used below its nominal "low-frequency cut-off point," in a region of rising frequency response. This region is deliberately extended by insertion of additional resistance R_p' until the time constant τ_4 is equal to the primary time constant τ_1. The maximum time constant that can be compensated for is $(N+1)L_1/R_p$. In the idealized condition, the maximum frequency response improvement factor obtainable is $N+1$ and the gain is unity.

Effect of loading on transformer-type compensator connected to single primary element. - When the transformer is connected to a source resistance R_1 and to a load resistance R_λ, the factors F_{34} and G_{34} are

$$F_{34} = \frac{(N+1)(1+\alpha)}{(1+\gamma_1 N+\gamma_2 N^2)} \tag{28b}$$

and

$$G_{34} = \left[\left(1 + \frac{R_1}{R_p+R_p'}\right)\left(1 + \frac{R_s}{R_\lambda}\right) + \frac{R_1}{R_\lambda}\right]^{-1} \tag{28c}$$

where α, γ_1, and γ_2 are functions of the circuit resistances and are given by

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$$\alpha = \frac{R_1}{R_p+R_p'} \frac{R_s+R_\lambda}{R_1+R_s+R_\lambda} \tag{28d}$$

$$\gamma_1 = \frac{2R_1}{(R_1+R_s+R_\lambda)} \tag{28e}$$

$$\gamma_2 = \frac{(R_1+R_p+R_p')}{(R_1+R_s+R_\lambda)} \tag{28f}$$

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The time constant τ_4 is unchanged by variations in source and load resistances and retains the value given by equation (28a).

The effect of a nonzero source resistance is to increase the frequency response improvement factor F_{34} and to decrease the gain factor G_{34} in the same proportion. The effect of a finite load resistance is to decrease both F_{34} and G_{34}. The effect of making $R_1 > 0$ and R_λ finite is to decrease both G_{34} and the product $F_{34}G_{34}$, so that the attenuation is increased without a proportionate increase in frequency response improvement factor. The requirement that $R_\lambda \gg N^2(R_1+R_p+R_p')$ imposes a severe limitation on the usefulness of the transformer-type compensator because the output power available becomes very small.

The characteristics of various transformers usable as compensators are listed in table III. Table IV gives, for compensators using these transformers and for various time constants τ_4, primary-element resistances R_1 and load resistances R_λ, values of frequency-response improvement factor F_{34}, the gain factor G_{34}, and the transconductance g_{34} representing the current through the load resistance per unit input emf.

Effect of loading on transformer-type compensator connected to two primary elements. - The attenuation of the primary element output due to the loading effect of the transformer primary can be eliminated by using two primary elements connected additively as shown in figure 14(c). Two adjacent primary elements of the same time constant are used to obtain the following relations:

$$\tau_4 = \frac{(N+1)L_1}{(R_1+R_p+R_p')} \tag{29a}$$

$$F_{34} = \frac{N+1}{1+\dfrac{N^2(R_1+R_p+R_p')}{R_1'+R_s+R_\lambda}} \tag{29b}$$

$$G_{34} = \frac{R_\lambda}{R_1{}'+R_s+R_\lambda} \tag{29c}$$

Table V gives, for various transformers, time constants τ_4, primary-element resistances R_1 and load resistances R_λ, the values of the frequency-response improvement factor F_{34}, the gain factor G_{34}, and the transconductance g_{34} representing the current through the load resistance per unit input emf. 2388

Summary of Compensating Systems

The various types of compensating element are summarized in table VI, and their more likely combinations are summarized in table VII, under certain idealized conditions noted in the tables. For various elements, table VI gives the design constants, the transfer function, the frequency-response amplitude ratio $|Y(j\omega)|$, the lower frequency limit ω_a, and the upper frequency limit ω_b (to be compared with the upper frequency limit $1/\tau_1$ of the primary element alone). For various arrangements of elements, table VII gives the resultant transfer function, the frequency-response amplitude ratio $|Y(j\omega)|$, the frequency-response improvement factor F, the lower and upper frequency limits ω_a and ω_b, and the gas temperature equivalent electric signal $E_{0,min}$ that yields a signal-to-noise ratio σ_4 of unity.

Off-Design Performance

In the previous sections the response characteristics of various compensating systems and of their components have been presented principally for those design conditions that provide best compensation. Deviations from these design conditions will be discussed herein for some of the simpler systems.

Mismatch between time constants of element and compensator. - Since the value of the time constant of the primary element is a function of the velocity and the density (or, equivalently, Mach number and static pressure) of the gas stream in which the element is being used (reference 4), the knowledge of the time constant is subject to some uncertainty. In addition, it is not always practical to make the necessary adjustments in the compensating system for a changed value of primary element time constant such as may arise when the element is replaced or when the operating conditions are altered. In practice, therefore, a certain amount of mismatch may exist between the primary element time constant and the time constant setting of the compensator. The effect of this mismatch is to destroy the flatness of the frequency response curve over the full normal range of compensation, although the flatness may be retained over a limited frequency range. If the primary element time constant drops below the compensator time constant, higher frequencies are amplified more than their normal amount; if the primary element time constant rises above the compensator time constant, higher frequencies are amplified less than their normal amount. An examination of the transfer function product $Y_1(p) \quad Y_4(p)$ will provide a measure of the effects of a mismatch wherein $\tau_1 \neq \tau_4$.

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For a compensating system (fig. 6) having the transfer function

$$Y_4(p) = G_4 \frac{1+\tau_4 p}{1+(\tau_4/F_4)p} \tag{30}$$

and connected to an element having a time constant τ_1, the frequency-response amplitude ratio $|Y_4(j\omega) \quad Y_1(j\omega)|$ is shown in figure 15(a) for various values of τ_4/τ_1, various values of F_4, and for F_4G_4 assumed equal to unity. Figure 15(b) shows the response to a step input. For very large values of F_4, the height of the initial rapid rise is proportional to the ratio τ_4/τ_1; the remainder of the change is described by a curve that is approximately exponential in form with a time constant τ_1.

For circuit elements (figs. 13 and 14) having a transfer function of the form

$$Y_{34}(p) = \mu_3 G_{34} \frac{1+\tau_4 p}{1+(\tau_4/F_{34})p} \tag{31}$$

or

$$Y_{34}(p) = G_{34} \frac{1+\tau_4 p}{1+(\tau_4/F_{34})p} \tag{32}$$

the ordinate of figure 15(a) must be multiplied by $\mu_3 G_{34} F_4$ and that of figure 15(b), by $G_{34}F_4$.

Effect of ambient temperature on transformer-type compensator. - In certain applications, one of which will be discussed under Example 2 of APPLICATIONS, the average level of the temperature signal from the primary element must be measured accurately with a potentiometer or equivalent indicator of very high input resistance while the same element is connected to a transformer-type compensator of the autotransformer kind shown in figure 14(b).

There then exists an error in average-value indication because the thermocouple, of resistance R_1, is loaded by the resistance $R_p + R_p'$ and because the transformer primary resistance R_p, being of copper, changes with the temperature of the transformer. The average correction to be applied to all indications is then

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$$e_1 - e_1' = \frac{e_1' \, R_1}{R_{p,0}+R_p'} \tag{33}$$

where

e_1 output emf from thermocouple

e_1' voltage measured by potentiometer

$R_{p,0}$ resistance of transformer primary at average operating temperature T_0

The fractional error $\Delta e_1'/e_1'$ to be expected in the indication e_1' because of a change ΔT_2 in the transformer temperature is

$$\frac{\Delta e_1'}{e_1'} = \alpha_0 \, \Delta T_2 \, \frac{R_1 R_{p,0}}{(R_1+R_p'+R_{p,0})(R_p'+R_{p,0})} \tag{34}$$

where α_0 is the temperature coefficient of resistance of R_p at temperature T_0 ($\alpha_0 = 0.0022/^{\circ}F$ for copper at 68° F).

It is seen that the error is approximately proportional to the element resistance R_1. Therefore it is important to keep this resistance as low as possible by the use of short low-resistance connecting leads. Table VIII shows the magnitude of error due to transformer temperature change that might be expected for various values of element resistance and of time constant setting for various transformers. The larger transformers have smaller errors.

APPLICATIONS

A specific problem of temperature measurement is generally treated by first establishing a tentative set of performance specifications, such as the sensibility $E_{0,min}$ (minimum signal to be detected) corresponding to the desired temperature sensibility $T_{0,min}$, the allowable signal-to-noise ratio σ, and the frequency range to be covered (between cut-off points ω_a and ω_b). Next it must be determined whether there exists a suitable combination of circuit elements, preferably as represented by commercially available equipment, that will approach the tentative specifications. Finally, there is the practical engineering problem of reproducing the combination selected in appropriate physical form to meet requirements of ruggedness, freedom from maintenance, ability to operate over a wide range of ambient temperature and of power supply voltage, long service life, compactness, and low weight. Particularly severe requirements may force the development of special amplifiers rather than allow the use of standard commercial equipment.

If control, as well as measurement, is to be performed by the apparatus, the dynamic characteristics of the control elements and of the process to be controlled, as represented (if they are linear) by their respective transfer functions, must also be considered. In such situations, it is conceivable that it may be not only permissible for the temperature signal to be overcompensated, but also very desirable.

Although the exact design of a complete compensating system will be determined by all the factors considered in this report, the following general statements will aid in the preliminary decision on the most promising line of attack:

(1) In general, the use of a transformer of high-quality, multiple-shielded, hum-bucking construction will be a valuable aid in improving the signal-to-noise ratio. Unless the transformer is used also as a compensator, the improvement in signal-to-noise ratio is accomplished at the expense of loss of d-c. response.

(2) The attainable frequency response improvement may be limited by the product of detector sensibility and over-all gain of the system.

(3) When an amplifier is used, the attainable frequency response may be limited by the equivalent input noise of the amplifier.

(4) Transformer-type compensators using typical transformers such as those listed in table III will not allow frequency response improvement factors much greater than 50.

(5) The potential performance of an RL or a transformer-type compensator may be seriously restricted if the element is called upon to feed a load resistance that is low compared with the output impedance of the compensator.

(6) The highest time constant that can be compensated by a transformer-type compensator is determined by the product of primary time constant and turns ratio and is of the order of several seconds.

(7) The highest time constant that can be compensated by an RL compensator is of the order of a few tenths of a second; the frequency response improvement attainable may be limited by resonances or other nonlinearities of the inductor.

(8) The highest time constant that can be compensated for and the frequency response improvement attainable are not limited by any intrinsic qualities of the RC compensator. 2388

Figure 16 shows three types of compensator constructed for use at the Lewis laboratory.

The following discussion will indicate the performance that may be expected from two representative combinations of apparatus that lend themselves, respectively, to (1) the indication of the alternating component of a temperature signal (Example 1, using an RC compensator); and (2) the indication of the average value as well as the alternating component of a temperature signal (Example 2, using a transformer-type compensator). Both examples given are characterized by the fact that most of the components used are standard commercial ones. Both examples employ a transformer as a valuable aid in improving the signal-to-noise ratio. In the first example, the transformer is used to match the low impedance of the primary element to the higher impedance of the amplifier, thus raising the signal level; in the second example, the transformer is used as a compensator that does not greatly attenuate the signal as the RC or RL compensators do.

Example 1. - For the assembly of apparatus represented by the block diagram of figure 17(a), the over-all gain factor is

$$G = |Y_1(j\omega)\ Y_2(j\omega)\ Y_3(j\omega)\ Y_4(j\omega)| = \mu_2\mu_3G_4 \tag{35}$$

and the frequency response improvement factor is $F = F_4$, as given in table I.

From equation (21a) and the fact that $E_{4,N} \leq E_{2,N}\mu_3$, the minimum signal that can be detected is either

$$E_{0,min} = \frac{E_{4,N}\sigma_4}{G} \leq \frac{E_{2,N}\sigma_4}{\mu_2G_4} \tag{36a}$$

or

$$E_{0,min} = \frac{G_4\delta}{\mu_2\mu_3} \tag{36b}$$

whichever of the two equations yields the greater value. Equation (36a) represents the limitation imposed by noise; equation (36b) represents the limitation imposed by detector sensibility. In equation (36), the important quantities are

$E_{0,min}$ sensibility of electric equivalent of temperature signal

$E_{2,N}$ equivalent input noise voltage of amplifier

σ_4 allowable signal-to-noise ratio at detector

G over-all gain factor

G_4 gain factor for combination of primary element and compensator, as given in table I

δ input voltage sensibility of detector

μ_2 transformer voltage gain, as given by equation (B5b)

μ_3 amplifier voltage gain in region of flat response

For the group of apparatus shown in figure 17(b), the following constants apply:

Primary element (thermocouple)

Time constant, τ_1, sec	0.1
Electric resistance, R_1, ohms	1.0
Thermoelectric power, Q, microvolts/°F	22

Transformer

Nominal primary impedance (manufacturer's value), ohms	2.5
Nominal secondary impedance (manufacturer's value), ohms . . .	50,000
Turns ratio, N	140
Low-frequency limit for 1 db drop in μ_2 (manufacturer's value), cps	20
Low-frequency cut-off (based on manufacturer's low-frequency limit), $\omega_{2,a}$, sec^{-1}	62
High-frequency limit for 1 db change in μ_2 (manufacturer's value), cps	20,000
D-c. resistance of primary winding, R_p, ohms	0.7
Maximum unbalanced direct current for 1.5 db drop in μ_2 (manufacturer's value), milliamp	5

Compensator: RC type, time constant τ_4

Amplifier

Input impedance, $R_{3,2}$, ohms	10^6
Output impedance, $R_{3,3}$, ohms	1500
Voltage gain in region of flat response, μ_3	100
Low-frequency limit for 2 percent drop in μ_3 (manufacturer's value), cps	10
Low-frequency cut-off (based on manufacturer's low-frequency limit), $\omega_{3,a}$, sec^{-1}	13
High-frequency limit for 2 percent change in μ_3 (manufacturer's value), cps	100,000
High-frequency cut-off with 0.02 microfarad shunt across output, for noise reduction ($\omega_{3,b}$ = 31,000), cps	5000
Noise level $E_{2,N}$ referred to input when output is shunted by 0.02 microfarad condenser, millivolts	0.01

Detector (cathode-ray oscilloscope)

Input impedance, R_5, ohms	2×10^6
Sensitivity, S_5, in. deflection/millivolt	0.03
Sensibility, δ_5, millivolt input	0.6

(The detector sensibility δ_5 is determined not only by oscilloscope characteristics, such as sharpness of focus and sensitivity, but also by the method of reading the deflections. The stated sensibility is based on the fact that the oscilloscope pattern will be photographed and read to the equivalent of 0.02 in. deflection on the oscilloscope screen.)

In this tabulation of apparatus, all components have been specified except the components R, R_C, and C of the compensator. For the design condition, $\tau_1 = \tau_4 = 0.1$ second and for a signal-to-noise ratio $\sigma_4 = 3$, table IX lists the sensibilities $E_{0,min}$ and $T_{0,min}(=E_{0,min}/Q)$, the sensitivity S, and the upper frequency limit $f_{4,b}$ for various combinations of R, R_C, and C. The method used to compute the data in table IX was (a) to assume various values of G_4 and to insert these values into equation (36) for computation of the sensibilities; (b) to assume various combinations of R_C and C, such that $R_C C = 0.1$ second and then, using the same values of G_4 as in step (a), to compute R and then F_4 from table I; and (c) to compute

$$f_{4,b} = \frac{\omega_{4,b}}{2\pi} = \frac{F_4}{2\pi\tau_1} \tag{37}$$

$$S = S_5 G_4 \mu_2 \mu_3 Q \tag{38}$$

As shown in table IX, the amplifier noise, rather than the detector sensibility, limits the minimum signal which can be detected.

Using the values of $S = 1$ inch per 100° F, $R_C = 100$ megohms, $C = 0.001$ microfarad, $R = 118{,}000$ ohms, and $f_{4,b} = 1430$ cycles per second, a minimum signal of 9° F can be detected over a frequency range of about 10 to 1400 cycles per second. Figure 17(c) shows the computed frequency response curves based upon manufacturer's values for the transformer and amplifier constants and the computed effect of variations in the thermocouple time constant τ_1. Experimental data points taken to check the computation are also shown in figure 17(c). The experimental points show that the manufacturer's statements of low-frequency cut-off points were conservative. The experimental data were obtained by using the analog circuit of figure 17(d) to simulate the thermocouple.

Since the resistances of both the primary element and the transformer primary winding are low (1 and 0.8 ohm, respectively), the net d-c. open-circuit output emf in the thermocouple circuit must be limited to 8.5 millivolts to avoid exceeding the transformer primary saturation current. Therefore, for temperature differences exceeding 380° F, a bucking circuit of the type shown in figure 17(e) would have to be used to suppress the average values of emf. The suppression of excessive direct current through the transformer primary could also be accomplished by use of the circuit of figure 17(f). The circuit of figure 17(f) is usable only at higher frequencies where the highest available size of capacitor can provide an impedance that is low compared with the transformer primary impedance.

The circuits of figures 17(e) and 17(f) would be equally applicable if the primary element were a temperature-sensitive resistor, such as a straight length of wire with current and potential leads.

The insertion of a 1-ohm resistance in series with the primary element would also serve to limit the transformer primary current at the expense of a loss in gain and a moderate increase in the lower cut-off frequency.

Example 2. - For the assembly of apparatus represented by the block diagram of figure 18(a), the time constant, frequency response improvement factor, and over-all gain factor are given by equation (28).

Since noise due to hum pickup can be made negligible by careful shielding and use of a high-quality multiple-shielded transformer, the minimum detectable signal will be determined by the detector sensibility.

For the group of apparatus shown in figure 18(b), the following constants apply: 2388

Primary element (thermocouple)

Time constant, τ_1, sec	1.0
Electric resistance, R_1, ohms	1.0
Thermoelectric power, Q, microvolts/$^{\circ}$F	22

Compensator (transformer type)

Nominal primary impedance (manufacturer's value), ohms	50
Nominal secondary impedance (manufacturer's value), ohms	100,000
D-c. resistance of primary winding, R_p, ohms	6
D-c. resistance of secondary winding, R_s, ohms	7000
Turns ratio, N	45
Added external primary circuit resistance, R_p', ohms	70
Time constant, τ_4, sec	1.0

Detector (d-c. amplifier and direct-writing recorder combination)

Input impedance, R_5, ohms	9×10^6
Sensitivity, S_5, millivolts/mm deflection	1.0
Cut-off frequency, cps	90
Sensibility, δ_5, millivolts (corresponding to 0.5 mm deflection)	0.5

The circuit required to provide reference-junction compensation for the thermocouple is shown in figure 18(c) (reference 9). If the primary element were a temperature-sensitive resistor, for example a straight length of wire with current and potential leads, a circuit such as shown in figure 18(d) could be used. This circuit, based on the multiple-bridge circuit analysis of reference 10, is designed to use a 0.02-ohm thermometer element (for example, a 0.008-in. diameter platinum wire, 0.25 in. long) to provide the same values of R_1 and Q as the thermocouple, and to limit to less than 2 percent the errors caused by changes in thermometer-element lead resistances as high as 1 ohm.

Figure 18(e) indicates the computed frequency response characteristics for operation both at the design point $(\tau_1 = \tau_4)$ and at off-design points. At the design point, the upper frequency limit is 2388 7 cycles per second $(\omega_b = 45)$ and is determined by the value $\tau_1/F = \tau_1/(N+1)$ rather than by the cut-off frequency of the recorder.

The experimentally determined responses to a step change and to an input of arbitrary shape are shown in figures 18(f) and 18(g), respectively, for the same ratios τ_1/τ_4 as covered in figure 18(e). The analog circuit of figure 17(d) was used to simulate the thermocouple.

Thus, the original 1.0-second time constant of the primary element is reduced by a factor of 46 to a value of approximately 0.02 second.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, January 4, 1952

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A_{dc}	attenuation at zero frequency
$\bar{B}_n$	coefficient of term in Fourier expansion of voltage signal representative of temperature
C	capacitance
c	constant
E	rms value of voltage
$\bar{E}$	amplitude of voltage sine wave
e	instantaneous value of voltage
F	frequency response improvement factor
f	frequency
G	gain factor
g	transconductance, output current per unit input emf
i	instantaneous value of current
j	$\sqrt{-1}$
L	self-inductance
M	mutual inductance
N	transformer turns ratio
n	number of term in Fourier expansion
p	symbolic operator d/dt
Q	emf per unit temperature change (= thermoelectric power, for a thermocouple)
R	resistance

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r	resistance ratio $(R_C+R)/R$ or $(R_L+R)/R_L$
S	sensitivity, detector deflection per unit input
T	temperature
$\overline{T}_0$	sinusoidal amplitude of impressed temperature variation
$\overline{T}_1$	sinusoidal amplitude of primary element temperature indication
t	time
t^*	time lag
X_C	reactance of capacitor
$Y(p)$	transfer function
$Y(j\omega)$	output-input sinusoidal voltage ratio for a system having a transfer function $Y(p)$
α	constant in expression for F of transformer-type compensator
β	feedback factor
β_{dc}	d-c. feedback factor
γ_1,γ_2	constants in expression for F of transformer-type compensator
δ	sensibility (minimum detectable signal)
ϵ	base of natural logarithms
θ	phase lag angle of impressed sine wave
μ	voltage gain
σ	signal-to-noise ratio
τ	time constant
φ	phase lag angle
ω	angular frequency

Subscripts:

a	low-frequency cut-off
b	high-frequency cut-off
N	noise
p	transformer primary
R	reference temperature
s	transformer secondary
λ	load
σ	source
0	temperature being measured
1	primary element
2	transformer preamplifier
3	amplifier
4	compensator
5	detector
34	amplifier-compensator combination

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APPENDIX B

TRANSFER FUNCTION OF TRANSFORMER

The transfer function of a transformer will be derived for those conditions that are pertinent to its applications as a preamplifier and as a compensator, as treated in this report. It is then possible to neglect the hysteresis and eddy current losses and the capacitances that ordinarily become important at higher power levels and higher frequencies. The elements that primarily enter into transformer performance as pertinent to this report are shown in figure 14(a), which shows a transformer connected to a source resistance R_σ and a load resistance R_λ.

The following differential equations hold:

$$e_1 = (R_1+R_p+L_1p)i_1 + Mpi_2 \tag{B1}$$

and

$$0 = (R_s+R_\lambda+L_2p)i_2 + Mpi_1 \tag{B2}$$

The transfer function $Y_2(p)$ of the transformer will be defined as the ratio of the voltage across the load resistance to the open-circuit emf of the source. The definition of $Y_2(p)$ thus includes the effects of the potential drops due to current flow through the source and through the transformer secondary. The following transfer function is obtained from the simultaneous solution of equations (B1) and (B2):

$$Y_2(p) = \frac{e_2}{e_1} = \frac{MR_\lambda p}{(R_s+R_\lambda)(R_1+R_p)+\left[(R_s+R_\lambda)L_1+(R_1+R_p)L_2\right]p+(L_1L_2-M^2)p^2} \tag{B3a}$$

where

$$e_2 = i_2R_\lambda$$

Equation (B3a) yields the frequency response function

$$Y_2(j\omega) = \frac{j\omega MR_\lambda}{(R_s+R_\lambda)(R_1+R_p)+j\omega\left[(R_s+R_\lambda)L_1+(R_1+R_p)L_2\right]-\omega^2(L_1L_2-M^2)} \tag{B4}$$

It is sufficient for the present analysis to assume $M^2 = L_1L_2$ and $N^2 = L_2/L_1$. Then equation (B3a) becomes

$$Y_2(p) = \mu_2 \frac{(1/\omega_{2,a})p}{1+(1/\omega_{2,a})p} \tag{B3b}$$

where

$$\frac{1}{\omega_{2,a}} = \frac{L_1}{R_1+R_p}\left(1+N^2 \frac{R_1+R_p}{R_s+R_\lambda}\right) \tag{B5a}$$

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$$\mu_2 = \frac{N}{1 + \frac{R_s}{R_\lambda} + N^2 \frac{R_1+R_p}{R_\lambda}} \tag{B5b}$$

The value of $|Y_2(j\omega)|$ in the region of flat frequency response is μ_2 and will be termed the "gain" of the transformer.

The angular frequency $\omega_{2,a}$ is the point at which $|Y_2(j\omega)| = \mu_2/\sqrt{2}$ and will be termed the "low-frequency cut-off" of the transformer.

If $R_\lambda >> R_s$, equation (B5b) becomes

$$\mu_2 \approx \frac{N}{1+\frac{N^2(R_1+R_p)}{R_\lambda}} \tag{B6}$$

Thus, if $R_\lambda >> R_s$, the voltage gain is approximately equal to $N/2$ if $R_\lambda/(R_1+R_p) = N^2$, and the voltage gain approaches N as $R_\lambda/(R_1+R_p)$ becomes much larger than N^2.

APPENDIX C

TRANSFER FUNCTION OF TRANSFORMER-TYPE COMPENSATOR

When the transformer is connected to a source of resistance R_1 and a load of resistance R_λ as shown in figure 14(b), the following differential equations hold:

$$i_1 = i_p + i_2 \tag{C1}$$

$$e_1 = R_1 i_1 + (R_p + R_p')i_p + L_1 p i_p - Mpi_2 \tag{C2}$$

$$e_1 = R_1 i_1 + R_s i_2 + R_\lambda i_2 + L_2 p i_2 - Mpi_p \tag{C3}$$

From the simultaneous solution of these equations, the following transfer function is obtained:

$$Y_{34}(p) = \frac{e_2}{e_1} = \frac{G_{34}(1 + \tau_4 p)}{1 + (\tau_4/F_{34})p + \gamma_3 p^2} \tag{C4a}$$

where

$$e_2 = i_2 R_\lambda \tag{C4b}$$

$$\tau_4 = \frac{L_1 + M}{R_p + R_p'} \tag{C4c}$$

$$G_{34} = \left[\left(1 + \frac{R_1}{R_p + R_p'}\right)\left(1 + \frac{R_s}{R_\lambda}\right) + \frac{R_1}{R_\lambda}\right]^{-1} \tag{C4d}$$

$$F_{34} = \frac{(N+1)(\alpha+1)}{1 + \gamma_1 N + \gamma_2 N^2} \tag{C4e}$$

$$\alpha = \frac{R_1}{R_p + R_p'} \, \frac{R_s + R_\lambda}{R_1 + R_s + R_\lambda} \tag{C4f}$$

$$\Upsilon_1 = \frac{2R_1}{R_1+R_s+R_\lambda} \tag{C4g}$$

$$\Upsilon_2 = \frac{R_1+R_p+R_p'}{R_1+R_s+R_\lambda} \tag{C4h}$$

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$$\Upsilon_3 = \frac{L_1L_2-M^2}{R_pR_\lambda G_{34}} \tag{C4i}$$

Equation (C4a) can be expressed in terms of frequency as

$$Y_{34}(j\omega) = \frac{G_{34}(1+j\omega\tau_4)}{1+j\omega\tau_4/F_{34}-\Upsilon_3\omega^2} \tag{C5}$$

It is sufficient for the present analysis to assume $M^2 = L_1L_2$ and $N^2 = L_2/L_1$. Then equations (C4a) and (C5) become

$$Y_{34}(p) \approx G_{34}\,\frac{1+\tau_4 p}{1+(\tau_4/F_{34})p} \tag{C6a}$$

$$Y_{34}(j\omega) \approx G_{34}\,\frac{1+j\omega\tau_4}{1+j\omega\tau_4/F_{34}} \tag{C7}$$

If the source and load resistances are assumed to be zero and infinite, respectively, the value of τ_4 is unchanged while the values of G and F become

$$G_{34} = 1 \tag{C8a}$$

$$F_{34} = N + 1 \tag{C8b}$$

The transfer function then becomes

$$Y_{34}(p) = \frac{1 + \tau_4 p}{1 + \frac{\tau_4}{N+1}\,p} \tag{C6b}$$

In the case of the transformer-type compensator that uses two primary elements, as shown in figure 14(c), the transfer function is also given by equation (C6a), but there are slight changes in the values of τ_4, G_{34}, and F_{34}:

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$$\tau_4 = \frac{L_1+M}{R_1+R_p+R_p'} \tag{C9a}$$

$$G_{34} = \frac{R_\lambda}{R_1+R_s+R_\lambda} \tag{C9b}$$

$$F_{34} = \frac{N+1}{1+\frac{N^2(R_1+R_p+R_p')}{R_1+R_s+R_\lambda}} \tag{C9c}$$

In the idealized case $R_1 = 0$ and $R_\lambda = \infty$, equations (C9a), (C9b), and (C9c) become, respectively,

$$\tau_4 = \frac{L_1+M}{R_p+R_p'} \tag{C10a}$$

$$G_{34} = 1 \tag{C10b}$$

$$F_{34} = N + 1 \tag{C10c}$$

and the transfer function remains as given by equation (C6b).

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TABLE I - EFFECT OF LOADING ON RC AND RL COMPENSATORS

Compensator	$R_\sigma = 0$ $R_\lambda = \infty$	$R_\sigma = 0$	$R_\lambda = \infty$	General case
Value of $1/G_4$				
RC	$\frac{R_C}{R} + 1$	$\frac{R_C}{\left(\frac{RR_\lambda}{R+R_\lambda}\right)} + 1$	$\frac{R_C+R_\sigma}{R} + 1$	$\frac{R_C + R_\sigma}{\left(\frac{RR_\lambda}{R+R_\lambda}\right)} + 1$
RL	$\frac{R}{R_L} + 1$	$\frac{R}{\left(\frac{R_L R_\lambda}{R_L+R_\lambda}\right)} + 1$	$\frac{R+R_\sigma}{R_L} + 1$	$\frac{R+R_\sigma}{\left(\frac{R_L R_\lambda}{R_L+R_\lambda}\right)} + 1$
Value of F_4				
RC	$\frac{R_C}{R} + 1$	$\frac{R_C}{\left(\frac{RR_\lambda}{R+R_\lambda}\right)} + 1$	$\frac{R_C}{R+R_\sigma} + 1$	$\frac{R_C}{\frac{RR_\lambda}{R+R_\lambda} + R_\sigma} + 1$
RL	$\frac{R}{R_L} + 1$	$\frac{\left(\frac{RR_\lambda}{R+R_\lambda}\right)}{R_L} + 1$	$\frac{R+R_\sigma}{R_L} + 1$	$\frac{\left[\frac{(R+R_\sigma)R_\lambda}{(R+R_\sigma)+R_\lambda}\right]}{R_L} + 1$

TABLE II - MINIMUM AMPLITUDE OF TEMPERATURE VARIATION THAT CAN BE MEASURED BECAUSE OF NOISE LIMITATION AT SIGNAL-TO-NOISE RATIO σ_4 EQUAL TO 3

Frequency response improvement factor F	Noise $E_{2,N}$ (microvolts)	Transformer gain μ_2	Signal sensibility $E_{0,min}$ (microvolts)	$T_{0,min}$ for chromel-alumel thermocouple (°F)
10	10	100	3	0.1
10	10	30	10	.5
10	100	100	30	1
10	100	30	100	5
100	10	100	30	1
100	10	30	100	5
100	100	100	300	14
100	100	30	1,000	45
1,000	10	100	300	14
1,000	10	30	1,000	45
1,000	100	100	3,000	140
1,000	100	30	10,000	450
10,000	10	100	3,000	140
10,000	10	30	10,000	450

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TABLE III - CHARACTERISTICS OF TRANSFORMERS USED IN REPRESENTATIVE

TRANSFORMER-TYPE COMPENSATORS

Compensator type	UTC/A 27	UTC/LS 39	ADC/AX 3270
Nominal[a] primary impedance, ohms	50	50	1
Primary inductance, L_1, henries	1.2	1.6	0.8
D-c. resistance of primary, R_p, ohms	9.5	5.5	0.16
Nominal[a] secondary impedance, ohms	100,000	100,000	1000
D-c. resistance of secondary, R_s, ohms	3000	7000	110
Turns ratio, N	45	45	32
Nominal[b] low-frequency cut-off point, cps	30	20	0.2
Nominal[b] high-frequency cut-off, cps	20,000	20,000	20
Weight, lb	0.5	3	13

[a]Manufacturer's stated value.

[b]Manufacturer's stated value for 2 decibels change in gain.

TABLE IV - EFFECT OF CONNECTING TRANSFORMER-TYPE COMPENSATOR WITH SINGLE PRIMARY ELEMENT TO PRIMARY ELEMENT RESISTANCE R_1 AND TO LOAD RESISTANCE R_λ

(a) Type UTC/A 27

R_λ (megohms)	R_1 (ohms)	τ_4, sec: 5.8			3			1			0.3			0.1		
		R_p', ohms: 0			9			45			183			540		
		F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)
10	1	51	0.91	0.09	49	0.95	0.10	47	0.98	0.10	44	0.99	0.10	41	1.0	0.10
	3	60	.75	.08	53	.86	.09	48	.95	.10	45	.98	.10	41	.99	.10
	10	92	.49	.05	71	.65	.06	54	.85	.08	47	.95	.10	42	.98	.10
1	1	50	0.91	0.91	47	0.95	0.95	42	0.98	0.98	33	0.99	0.99	22	1.0	1.00
	3	59	.75	.75	51	.86	.86	43	.95	.95	34	.98	.98	22	.99	.99
	10	88	.49	.49	67	.65	.65	48	.85	.85	35	.95	.95	22	.98	.98
0.1	1	42	0.88	8.8	35	0.92	9.2	21	0.95	9.5	10	0.97	9.7	4	0.98	9.8
	3	50	.74	7.4	38	.84	8.4	22	.92	9.2	10	.95	9.5	4	.96	9.6
	10	70	.47	4.7	46	.63	6.3	25	.82	8.2	10	.92	9.2	4	.95	9.5
0.01	1	19	0.70	70	12	0.73	73	5	0.75	75	2	0.77	77			
	3	20	.58	58	12	.66	66	5	.75	75	2	.75	75			
	10	23	.38	38	13	.50	50	5	.65	65	2	.73	73			
0.001	1	8	0.23	230	4	0.24	240	2	0.25	250						
	3	8	.19	190	4	.21	210	2	.24	240						
	10	8	.12	120	5	.16	160	2	.21	210						

TABLE IV - EFFECT OF CONNECTING TRANSFORMER-TYPE COMPENSATOR WITH SINGLE PRIMARY ELEMENT TO PRIMARY ELEMENT RESISTANCE R_1 AND TO LOAD RESISTANCE R_λ - Continued

(b) Type UTC/LS 39

R_λ (megohms)	R_1 (ohms)	T_4, sec																	
		14			10			3			1			0.3			0.1		
		R_p', ohms																	
		0			2			19			69			249			735		
		F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)
10	1	55	0.84	0.08	52	0.88	0.10	48	0.96	0.10	46	0.99	0.10	44	1.0	0.10	40	1.0	0.10
	3	72	.64	.06	65	.71	.07	52	.90	.09	47	.96	.10	44	.99	.10	40	1.0	.10
	10	133	.34	.03	108	.43	.04	64	.72	.07	51	.88	.09	46	.96	.10	40	1.0	.10
1	1	54	0.83	0.83	51	0.88	0.88	48	0.95	0.95	40	0.98	0.98	31	0.99	0.99	18	0.99	0.99
	3	71	.63	.63	62	.71	.71	49	.89	.89	41	.95	.95	31	.98	.98	18	.98	.98
	10	129	.33	.33	102	.43	.43	60	.71	.71	45	.87	.87	32	.95	.95	18	.98	.98
0.1	1	49	0.78	7.8	45	0.82	8.2	32	0.90	9.0	19	0.93	9.3	8	0.93	9.3	3	0.92	9.2
	3	62	.60	6.0	54	.66	6.6	34	.83	8.3	20	.90	9.0	8	.92	9.2	3	.92	9.2
	10	103	.32	3.2	81	.40	4.0	37	.67	6.7	20	.82	8.2	8	.90	9.0	3	.92	9.2
0.01	1	31	0.49	49	26	0.52	52	12	0.58	56	5	0.59	59	2	0.59	59			
	3	35	.38	38	29	.42	42	12	.52	52	5	.57	57	2	.58	58			
	10	47	.20	20	35	.25	25	13	.42	42	5	.52	52	2	.56	56			
0.001	1	21	0.10	100	17	0.11	110	6	0.12	120	2	0.12	120						
	3	22	.08	80	21	.09	90	7	.11	110	2	.12	120						
	10	28	.04	40	20	.05	50	7	.09	90	2	.11	110						

TABLE IV - EFFECT OF CONNECTING TRANSFORMER-TYPE COMPENSATOR WITH SINGLE PRIMARY ELEMENT TO PRIMARY ELEMENT RESISTANCE R_1 AND TO LOAD RESISTANCE R_λ - Concluded

(c) Type ADO/AX 3270

R_λ (megohms)	R_1 (ohms)	T_4, sec																	
		160			30			10			3			1			0.3		
		R_p', ohms																	
		0			0.7			2.5			8.8			26			88		
		F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)	F_{34}	G_{34}	g_{34} (micromhos)
10	0.1	53	0.62	0.06	37	0.90	0.09	34	0.96	0.10	33	0.99	0.10	33	0.99	0.10	33	1.00	0.10
	1	240	.14	.01	70	.47	.05	46	.72	.07	37	.90	.90	34	.96	.10	33	.99	.10
	10				410	.08	.01	160	.21	.02	71	.47	.05	45	.73	.07	36	.90	.09
1	0.1	53	0.62	0.62	37	0.90	0.90	34	0.96	0.96	33	0.99	0.99	32	1.00	1.00	30	1.00	1.00
	1	240	.14	.14	70	.47	.47	46	.72	.72	36	.90	.90	33	.96	.96	30	.99	.99
	10				410	.08	.08	150	.21	.21	68	.47	.47	43	.73	.73	34	.90	.90
0.1	0.1	53	0.62	6.2	37	0.90	9.0	32	0.96	9.6	31	0.99	9.9	26	1.00	10	18	1.00	10
	1	230	.14	1.4	70	.47	4.7	44	.72	7.2	33	.90	9.0	27	.96	9.6	18	.99	9.9
	10				360	.08	.8	140	.21	2.1	58	.47	4.7	33	.73	7.3	19	.90	9.0
0.01	0.1	52	0.61	61	35	0.90	90	27	0.95	95	17	0.98	98	9	0.94	94	3	0.99	99
	1	210	.14	14	61	.47	47	33	.71	71	19	.89	89	9	.95	95	3	.98	98
	10				200	.08	8	70	.21	21	25	.47	47	10	.72	72	3	.89	89
0.001	0.1	42	0.56	560	29	0.81	810	9	0.87	870	4	0.87	870						
	1	110	.13	130	32	.43	430	10	.65	650	4	.65	650						
	10				36	.07	70	12	.19	190	4	.19	190						

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TABLE V - EFFECT OF CONNECTING TRANSFORMER-TYPE COMPENSATOR WITH TWO PRIMARY ELEMENTS TO PRIMARY ELEMENT RESISTANCE R_1 AND LOAD RESISTANCE R_λ

(a) Type UTC/A 27

R_λ (megohms)	R_1 (ohms)	T_4, sec										G_{34}	g_{34} (micromhos)
		5.3		3.0		1.0		0.3		0.1			
		R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}		
10	1	0	46	6.5	46	54	45	182	45	550	41	1.0	0.1
	3			4.5	46	52	45	180	45	548	41		
	10					45	45	173	45	541	41		
1	1	0	45	6.5	44	54	41	182	34	550	22	1.0	1.0
	3			4.5	44	52	41	180	34	548	22		
	10					45	41	173	34	541	22		
0.1	1	0	38	6.5	34	54	22	182	10	550	4	0.97	9.7
	3			4.5	34	52	22	180	10	548	4		
	10					45	22	173	10	541	4		
0.01	1	0	18	6.5	12	54	5	182	2			0.77	77
	3			4.5	12	52	5	180	2				
	10					45	5	173	2				
0.001	1	0	7	6.5	5	54	2					0.25	250
	3			4.5	5	52	2						
	10					45	2						

TABLE V - EFFECT OF CONNECTING TRANSFORMER-TYPE COMPENSATOR WITH

TWO PRIMARY ELEMENTS TO PRIMARY ELEMENT RESISTANCE R_1 AND

LOAD RESISTANCE R_λ - Continued

(b) Type UTC/LS 39

R_λ (megohms)	R_1 (ohms)	T_4, sec												G_{34}	g_{34} (micromhos)
		11.4		10		3.0		1.0		0.3		0.1			
		R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}		
10	1	0	46	1.0	46	23	46	73	46	242	44	739	40	1.0	0.1
	3					21	46	71	46	240	44	737	40		
	10					14	46	64	46	233	44	730	40		
1	1	0	45	1.0	45	23	44	73	40	242	31	739	19	1.0	1.0
	3					21	44	71	40	240	31	737	19		
	10					14	44	64	40	233	31	730	19		
0.1	1	0	41	1.0	40	23	32	73	19	242	8	739	3	0.93	9.3
	3					21	32	71	19	240	8	737	3		
	10					14	32	64	19	233	8	730	3		
0.01	1	0	26	1.0	25	23	12	73	5	242	2			0.59	59
	3					21	12	71	5	240	2				
	10					14	12	64	5	233	2				
0.001	1	0	18	1.0	16	23	6	73	2					0.12	120
	3					21	6	71	2						
	10					14	6	64	2						

TABLE V - EFFECT OF CONNECTING TRANSFORMER-TYPE COMPENSATOR WITH

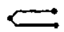
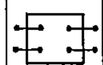
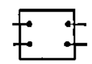
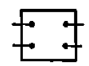
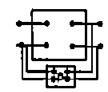
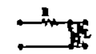
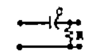
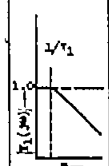
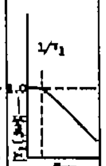
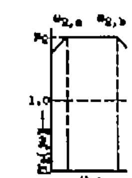
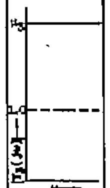
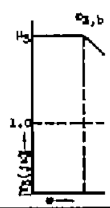
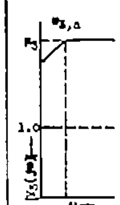
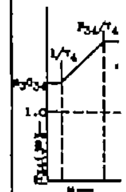
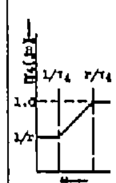
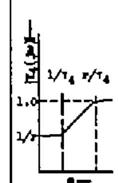
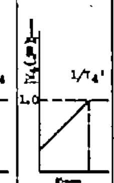
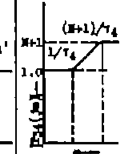
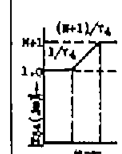
TWO PRIMARY ELEMENTS TO PRIMARY ELEMENT RESISTANCE R_1 AND

LOAD RESISTANCE R_λ - Concluded

(c) Type ADC/AX 3270

R_λ (megohms)	R_1 (ohms)	τ_4, sec												G_{34}	g_{34} (micromhos)
		98		30		10		3.0		1.0		0.3			
		R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}	R_p'	F_{34}		
	0.1	0	33	0.88	33	2.5	33	8.4	33	26.3	33	88	33		
10	1.0					1.6	33	7.5	33	25.4	33	87	33	1.0	0.1
	10									16.4	33	78	33		
	0.1	0	33	0.88	33	2.5	33	8.4	33	26.3	32	88	30		
1	1.0					1.6	33	7.5	33	25.4	32	87	30	1.0	1.0
	10									16.4	32	78	30		
	0.1	0	33	0.88	33	2.5	32	8.4	30	26.3	23	88	18		
0.1	1.0					1.6	32	7.5	30	25.4	23	87	18	1.0	10
	10									16.4	23	78	18		
	0.1	0	32	0.88	30	2.5	26	8.4	18	26.3	9	88	3		
0.01	1.0					1.6	26	7.5	18	25.4	9	87	3	0.99	99
	10									16.4	9	78	3		
	0.1	0	26	0.88	18	2.5	9	8.4	4	26.3	1				
0.001	1.0					1.6	9	7.5	4	25.4	1			0.89	890
										16.4	1				

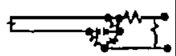
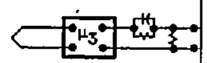
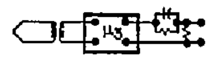
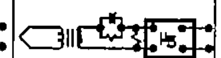
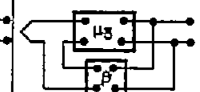
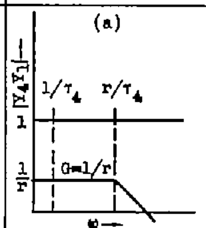
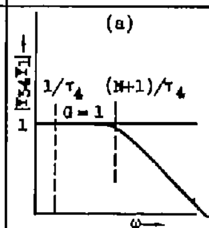
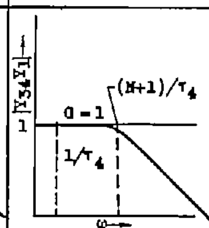
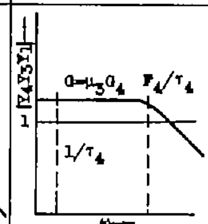
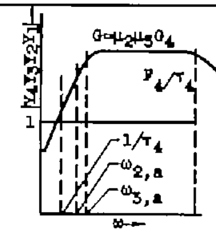
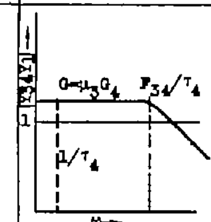
TABLE VI - SUMMARY OF COMPENSATING ELEMENTS

	PRIMARY ELEMENTS		TRANSFORMER	AMPLIFIERS				COMPENSATORS				
Component	Thermocouple	Resistance thermometer		D-c., no low- or high-frequency cut-off	D-c., high-frequency cut-off	A-c., low-frequency cut-off	Negative feed-back, with compensating network	RC type	RL type	Differentiating type	Transformer-type, single element	Transformer-type, double element
												
Design constants	τ_1 $Q=\frac{e_1}{T_1-T_R}$ $[E_0=Q(T_0-T_R)]$	τ_1 $Q=\frac{e_1}{T_1-T_R}$ $[E_0=Q(T_0-T_R)]$	$\omega_{2,a}$ $\omega_{2,b}$ N μ_2	μ_3	μ_3 $\omega_{3,b}$	μ_3 $\omega_{3,a}$	τ_4 μ_3 β_{d4} $F_{34}=1/G_{34}$ $=1-\beta_{d4}\mu_3$	$\tau_4=R_0C$ $r=(R_0/R)+1$ $F_4=r$ $G_4=1/r$ (a)	$\tau_4=L/R_L$ $r=(R/R_L)+1$ $F_4=r$ $G_4=1/r$ (a)	$\tau_4'=RC$ $F_4=\tau_1/\tau_4'$ (a)	$\tau_4=(N+1)\frac{L}{R_p+R_p'}$ $F_{34}=N+1$ $G_{34}=1$ (a)	$\tau_4=(N+1)\frac{L}{R_p+R_p'}$ $F_{34}=N+1$ $G_{34}=1$ (a)
Transfer function	$Y_1=\frac{1}{1+\tau_1 p}$	$Y_1=\frac{1}{1+\tau_1 p}$	$Y_2=\mu_2\frac{(1/\omega_{2,a})p}{[1+(1/\omega_{2,a})p][1+(1/\omega_{2,b})p]}$	$Y_3=\mu_3$	$Y_3=\mu_3\frac{1}{1+(1/\omega_{3,b})p}$	$Y_3=\frac{\mu_3(1/\omega_{3,a})p}{1+(1/\omega_{3,a})p}$	$Y_{34}=\mu_3 G_{34}\frac{1+\tau_4 p}{1+\frac{\tau_4 p}{F_{34}}}$	$Y_4=\frac{1}{r}\frac{1+\tau_4 p}{1+\left(\frac{\tau_4}{r}\right)p}$	$Y_4=\frac{1}{r}\frac{1+\tau_4 p}{1+\left(\frac{\tau_4}{r}\right)p}$	$Y_4=\frac{\tau_4' p}{1+\tau_4' p}$	$Y_{34}=\frac{1+\tau_4 p}{1+\frac{\tau_4 p}{N+1}}$	$Y_{34}=\frac{1+\tau_4 p}{1+\frac{\tau_4 p}{N+1}}$
Frequency response								(a) 	(a) 	(a) 	(a) 	(a) 
Lower limit of frequency range	0	0	$\omega_{2,a}$	0	0	$\omega_{3,a}$	$1/\tau_4$	$1/\tau_4$	$1/\tau_4$	0	$1/\tau_4$	$1/\tau_4$
Upper limit of frequency range	$1/\tau_1$	$1/\tau_1$	$\omega_{2,b}$	∞	$\omega_{3,b}$	∞	F_{34}/τ_4	r/τ_4 (a)	r/τ_4 (a)	$1/\tau_4'$ (a)	$(N+1)/\tau_4$ (a)	$(N+1)/\tau_4$ (a)

[a]For conditions of negligible source resistance and load admittance.

TABLE VII - SUMMARY OF COMPENSATING SYSTEMS

[For condition $\tau_4 = \tau_1$]

System	Primary element and RC or RL compensator	Primary element and transformer-type compensator	Double primary element and transformer-type compensator	Primary element, RC or RL type compensator, and d-c. amplifier	Primary element, transformer, RC or RL type compensator, and amplifier with low-frequency cut-off (normal order)	Primary element, transformer, RC or RL type compensator, and amplifier with low-frequency cut-off (inverted order)	Primary element and negative feedback amplifier with compensating network in feedback loop
Typical components							
Transfer function ($R_1=0$, $R_\lambda=\infty$)	$Y_4Y_1 = \frac{1}{r}\,\frac{1}{1+\left(\frac{\tau_4}{r}\right)p}$	$Y_{34}Y_1 = \frac{1}{1+\frac{\tau_4 p}{N+1}}$	$Y_{34}Y_1 = \frac{1}{1+\frac{\tau_4 p}{N+1}}$	$Y_4Y_3Y_1 = \mu_3G_4\,\frac{1}{1+\frac{\tau_4 p}{F_4}}$	$Y_4Y_3Y_2Y_1 = \mu_2\mu_3G_4\,\frac{p/\omega_{2,a}}{1+p/\omega_{2,a}}\,\frac{p/\omega_{3,a}}{1+p/\omega_{3,a}}\,\frac{1}{1+\frac{\tau_4 p}{F_4}}$		$Y_{34}Y_1 = \mu_3G_{34}\,\frac{1}{1+\frac{\tau_4 p}{F_{34}}}$
Frequency response of system							
Low-frequency cut-off ω_a	None	None	None	None	$\approx\omega_{2,a}+\omega_{3,a}$		None
High-frequency cut-off ω_b	r/τ_4 (a)	$(N+1)/\tau_4$ (a)	$(N+1)/\tau_4$ (a)	F_4/τ_4	F_4/τ_4		F_{34}/τ_4
Minimum input signal for $\sigma_4=1$; $E_{0,min}$	0	0	0	$< E_{2,N}/G_4$	$\le E_{2,N}/\mu_2G_4$	$E_{2,N}/\mu_2G_4$	$E_{2,N}/G_{34}$

[a]For condition of negligible primary element resistance and load admittance.

TABLE VIII - EFFECT OF AMBIENT TEMPERATURE ON D-C. OUTPUT OF TRANSFORMER-TYPE COMPENSATOR

(a) Type UTC/A 27

R_1 (ohms)	τ_4			
	3.0	1.0	0.3	0.1
	Percentage error for 100° F temperature change			
0.3	0.2	0.02	0.00	0.00
1.0	.6	.07	.01	.00
3.0	1.5	.2	.02	.00
10	3.8	.7	.06	.00

(b) Type UTC/LS 39

R_1 (ohms)	τ_4				
	10	3.0	1.0	0.3	0.1
	Percentage error for 100° F temperature change				
0.3	0.6	0.06	0.00	0.00	0.00
1.0	1.9	.2	.02	.00	.00
3.0	4.6	.6	.06	.01	.00
10	9.3	1.4	.2	.02	.00

(c) Type ADC/AX 3270

R_1 (ohms)	τ_4				
	30	10	3	1	0.3
	Percentage error for 100° F temperature change				
0.3	0.9	0.1	0.01	0.00	0.00
1.0	1.9	.4	.04	.00	.00
3.0	2.7	.7	.1	.01	.00
10	3.2	1.2	.2	.04	.00

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TABLE IX - PERFORMANCE OF COMPENSATING SYSTEM OF FIGURE 17(b) FOR VARIOUS COMBINATIONS OF R, R_C, AND C OF COMPENSATOR

[Time constant τ_4, 0.1 sec; signal-to-noise ratio σ_4, 3]

Sensitivity	Sensibility (due to amplifier noise)		Sensibility (due to detector sensitivity)		R_C = 1 megohm C = 0.1 mfd		R_C = 10 megohms C = 0.01 mfd		R_C = 100 megohms C = 0.001 mfd	
S (in./100° F)	$E_{0,min}$ (microvolts)	$T_{0,min}$[a] (°F)	$E_{0,min}$ (microvolts)	$T_{0,min}$[a] (°F)	R (ohms)	$f_{4,b}$ (cps)	R (ohms)	$f_{4,b}$ (cps)	R (ohms)	$f_{4,b}$ (cps)
30	6.7	0.3	1.3	0.06	35,700	47	418,000	48	---------	----
10	20	0.9	4	0.2	11,000	127	120,000	140	2,570,000	145
3	67	3	13	0.6	3,300	330	34,000	460	417,000	480
1	200	9	40	1.8	1,100	610	11,000	1270	118,000	1430
0.3	670	30	135	6.0	330	870	3,300	3300	34,000	4600

[a]Chromel-alumel thermocouple.

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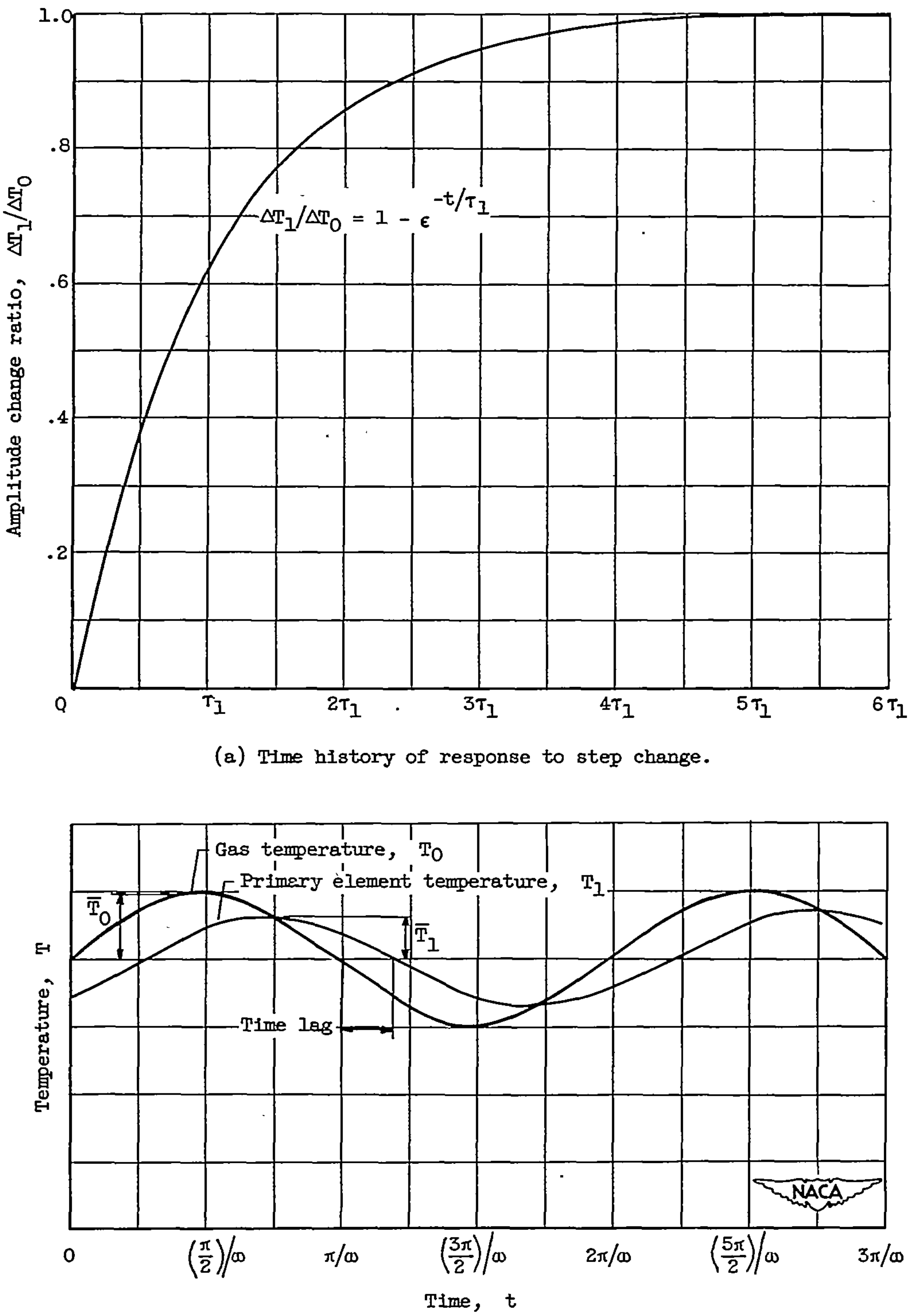

(a) Time history of response to step change.

(b) Time history of response to sinusoidal variation.

Figure 1. - Response of primary element.

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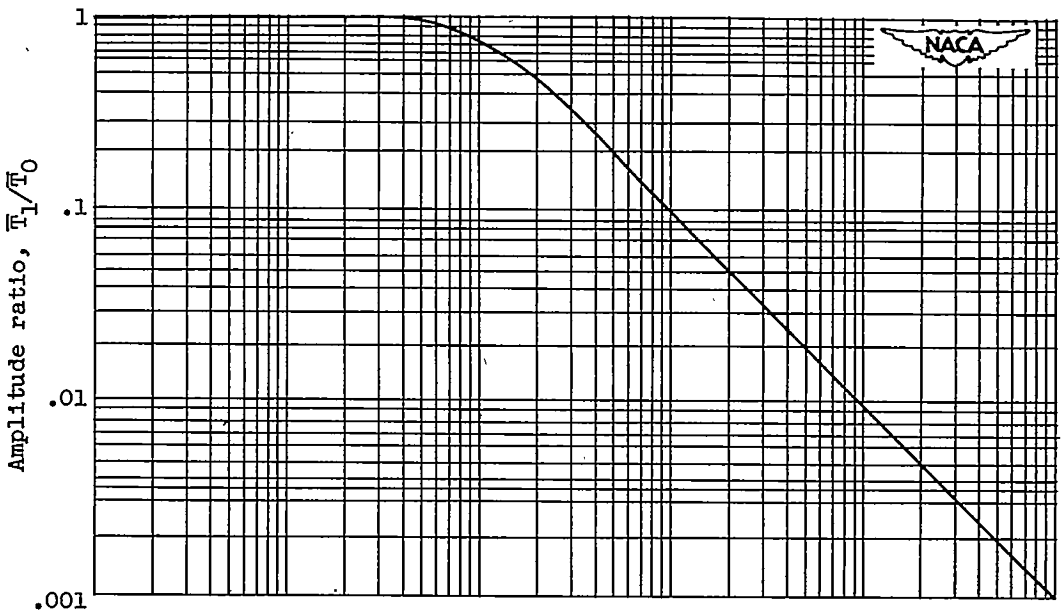
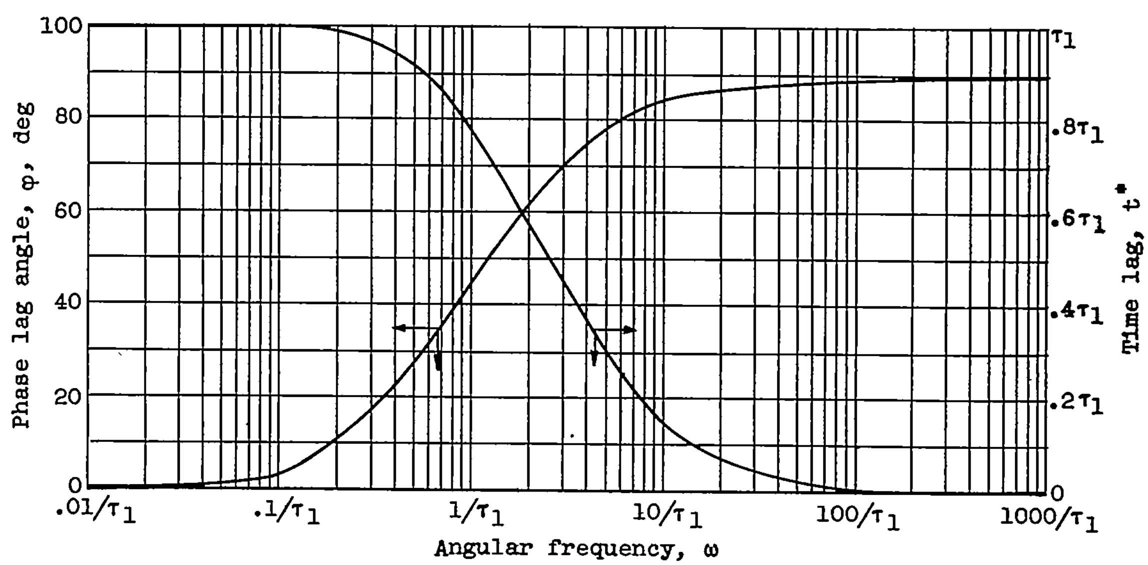


(c) Frequency-response relation; amplitude ratio.



(d) Frequency-response relation; phase and time lag.

Figure 1. - Concluded. Response of primary element.

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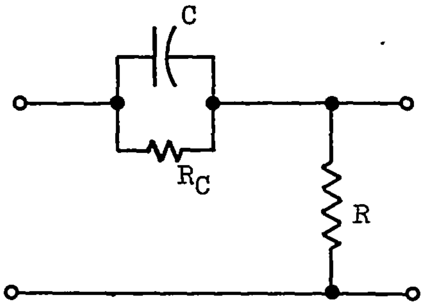
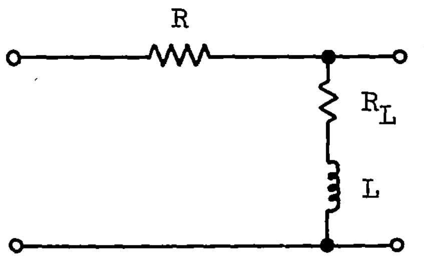


$$Y_4(p) = \frac{1}{r}\,\frac{1 + \tau_4 p}{1 + (\tau_4/r)p}$$

$$\tau_4 = R_C C$$

$$r = R_C/R + 1$$

(a) RC type.

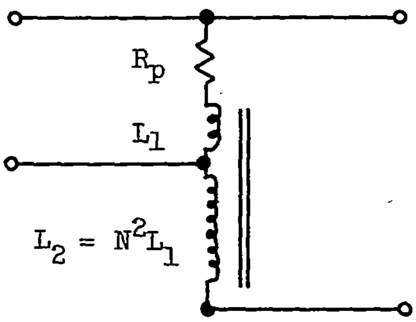


$$Y_4(p) = \frac{1}{r}\,\frac{1 + \tau_4 p}{1 + (\tau_4/r)p}$$

$$\tau_4 = L/R_L$$

$$r = R/R_L + 1$$

(b) RL type.



$$Y_{34}(p) = \frac{1 + \tau_4 p}{1 + \frac{\tau_4 p}{N+1}}$$

$$\tau_4 = (N+1)\ L_1/R_p$$

(c) Transformer type.

Figure 2. - Basic compensator networks and their transfer functions.

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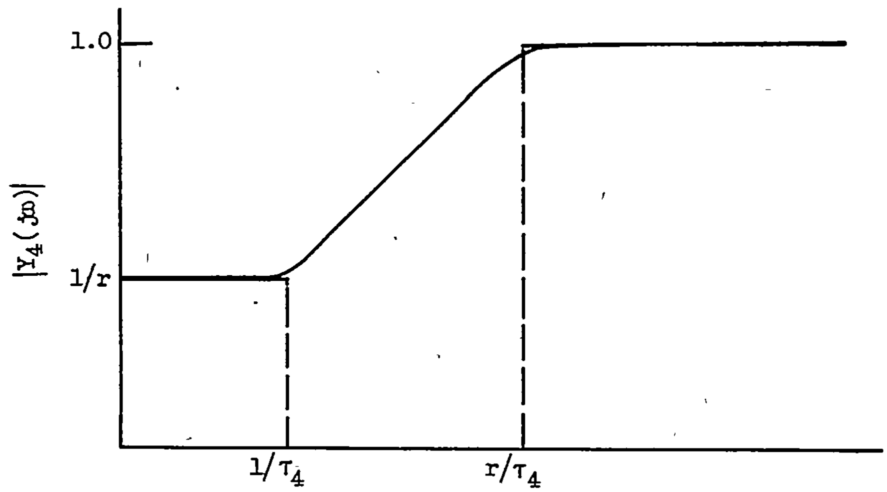
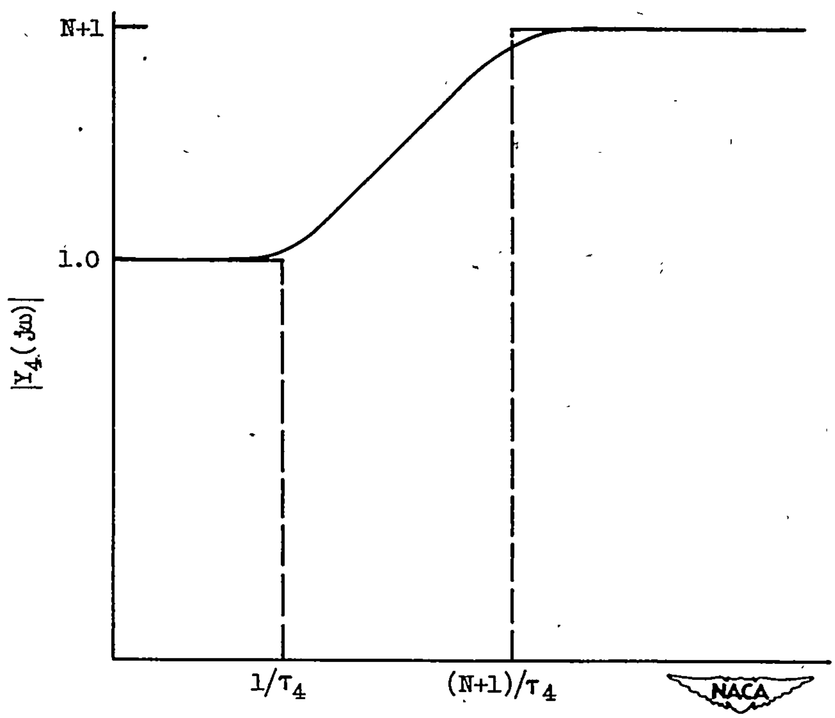
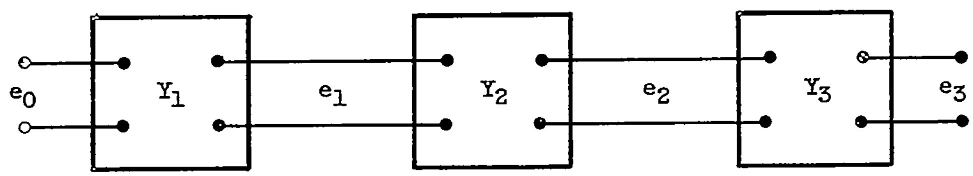


(d) Value of $|Y(j\omega)|$ as function of frequency for RC and RL compensators. (Log-log scale.)



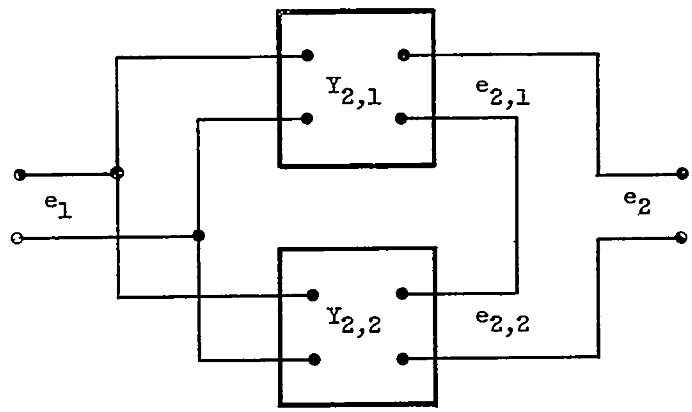
(e) Value of $|Y(j\omega)|$ as function of frequency for transformer-type compensator. (Log-log scale.)

Figure 2. - Concluded. Basic compensator networks and their transfer functions.



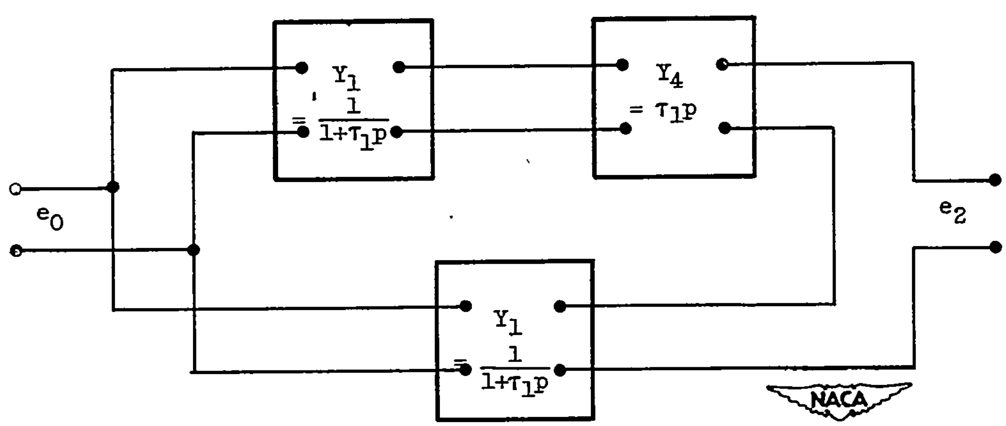
$$e_3 = Y_3e_2 = Y_3Y_2e_1 = Y_3Y_2Y_1e_0$$

(a) Cascaded system.



$$e_2 = e_{2,1} + e_{2,2} = (Y_{2,1} + Y_{2,2})\ e_1 = Y_2e_1$$

(b) Additive system.



$$e_2/e_0 = Y_1 + Y_1Y_4 = 1$$

(c) Example of combined cascaded and additive systems.

Figure 3. - Transfer function composition for linear systems.

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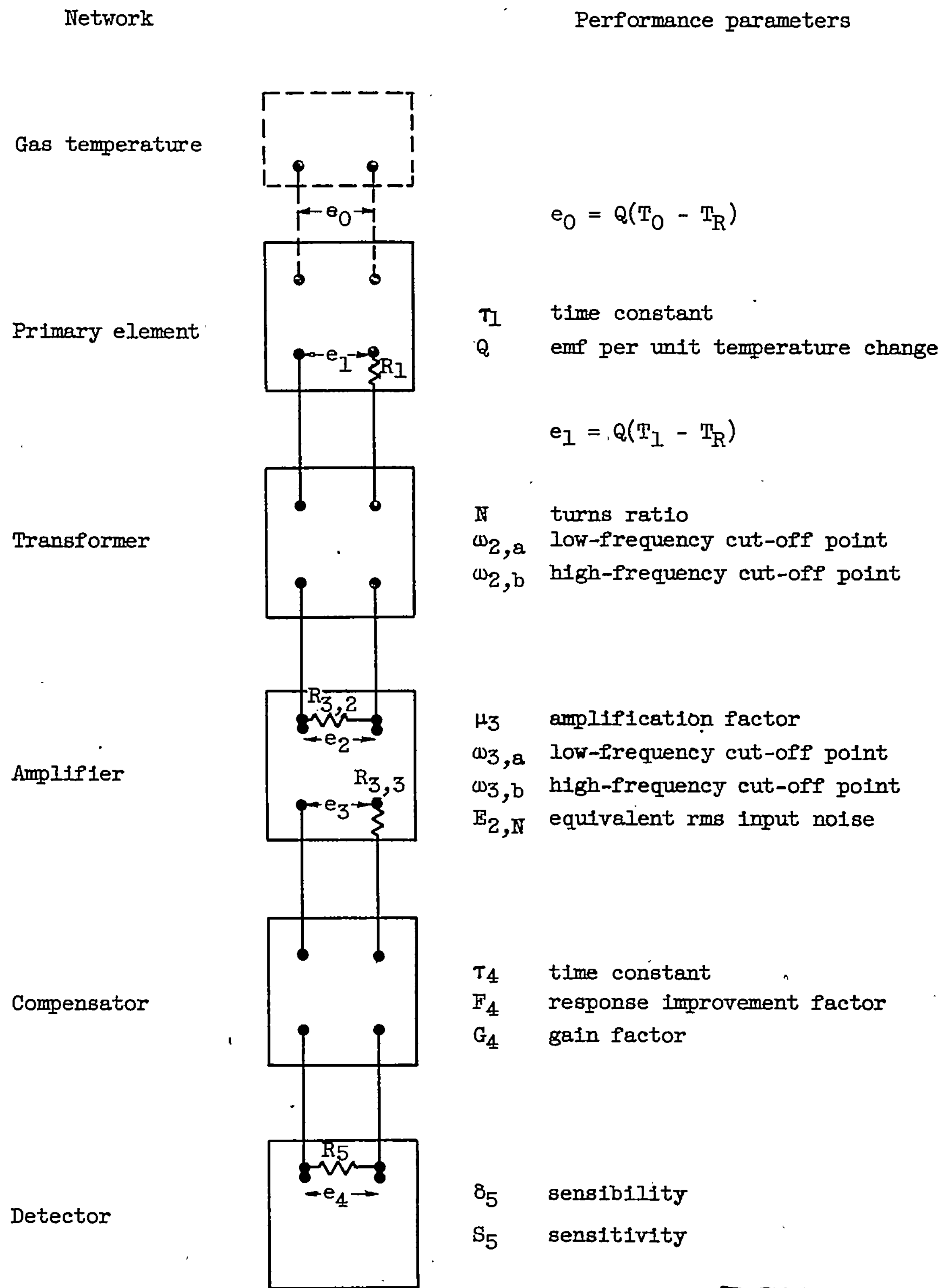
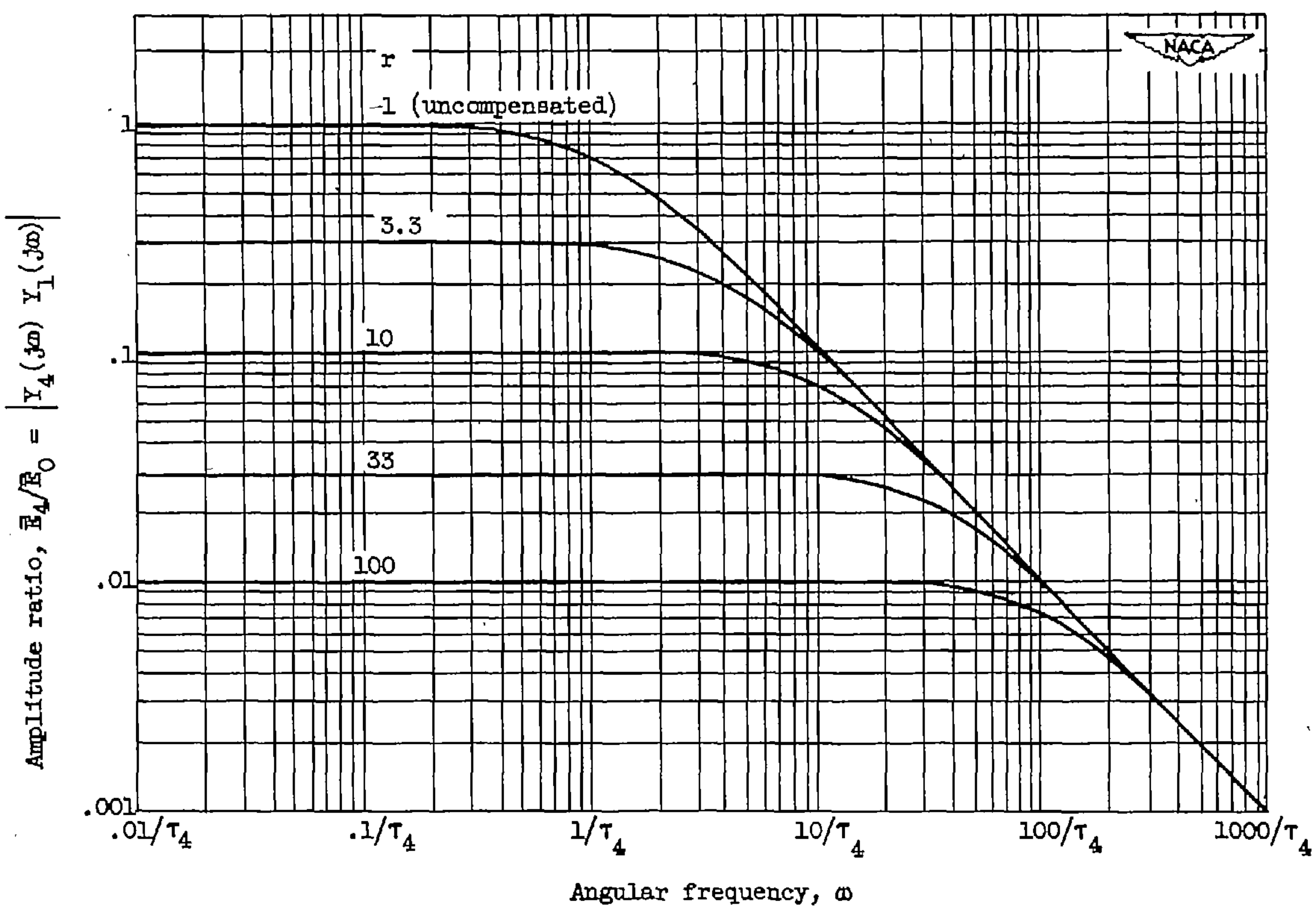
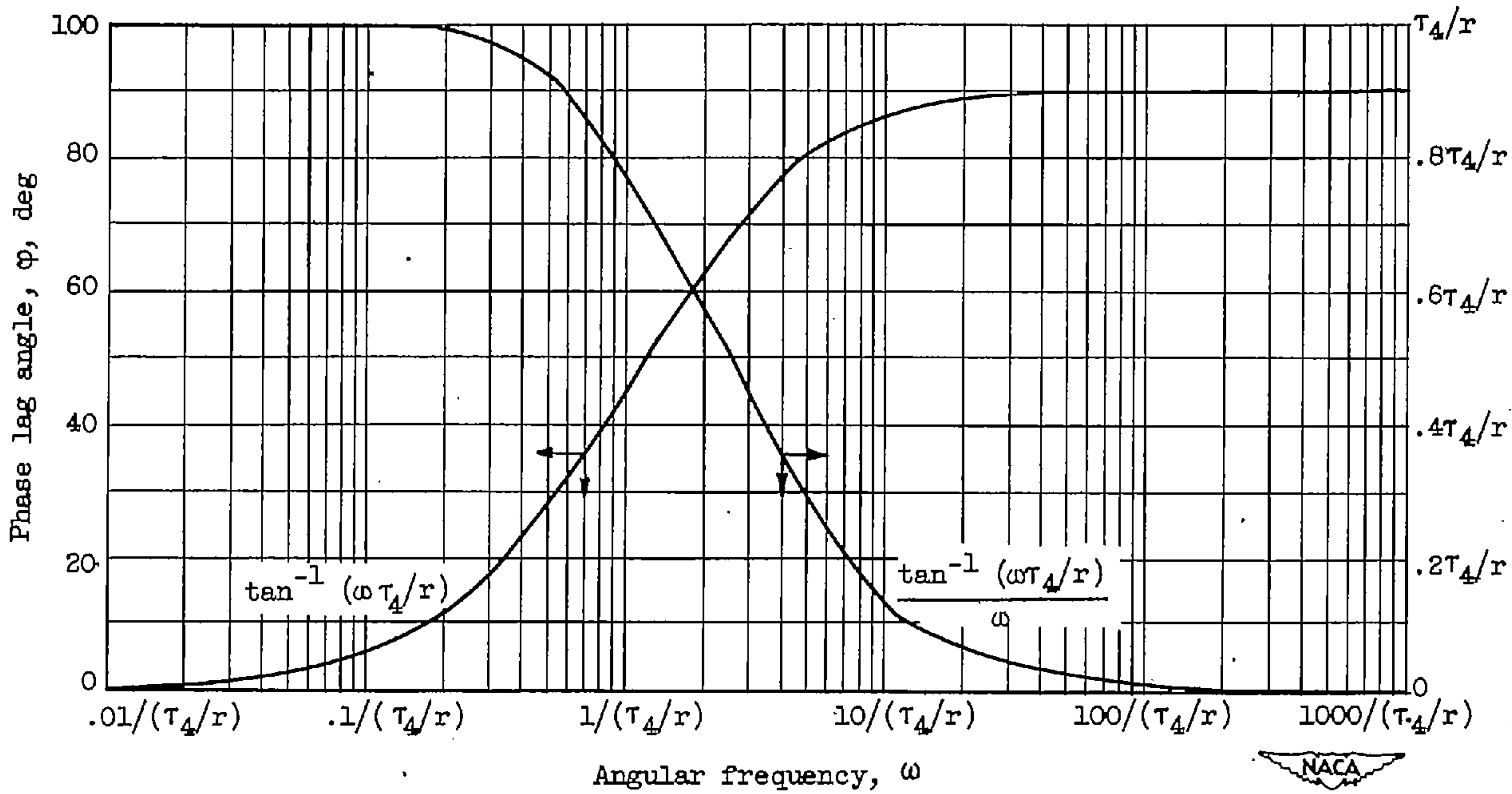


Figure 4. - Typical block diagram for definition of terminology.



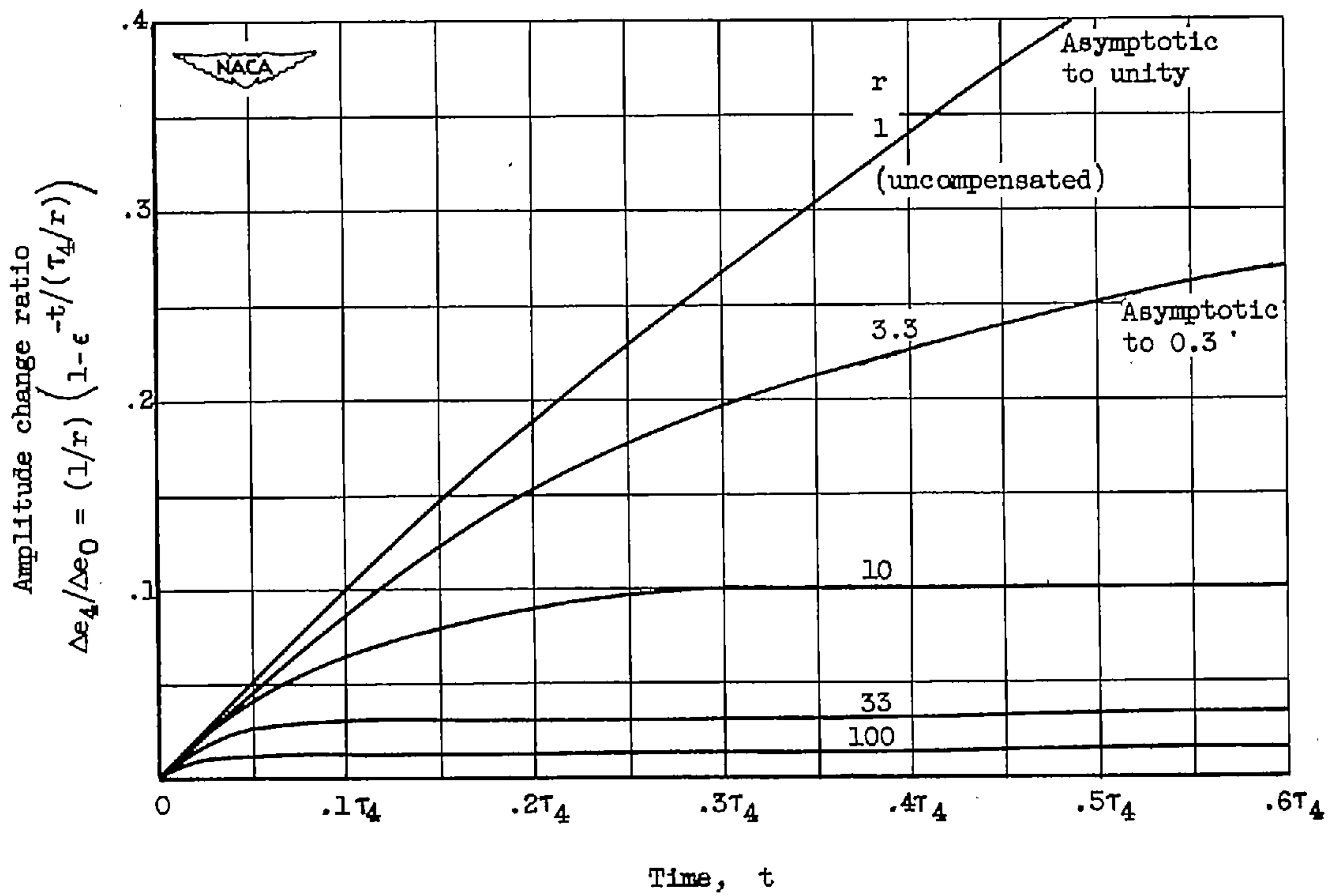
(a) Frequency-response relation; amplitude ratio.

Figure 5. - Response of primary element with RC or RL compensator. Primary element resistance and compensator load admittance assumed negligible. $\tau_4 = \tau_1$.



(b) Frequency-response relation; phase lag and time lag.

Figure 5. - Continued. Response of primary element with RC or RL compensator. Primary element resistance and compensator load admittance assumed negligible. $\tau_4 = \tau_1$.



(c) Time history of response to step change.

Figure 5. - Concluded. Response of primary element with RC or RL compensator. Primary element resistance and compensator load admittance assumed negligible. $\tau_4 = \tau_1$.

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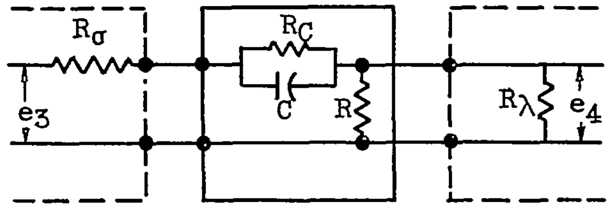
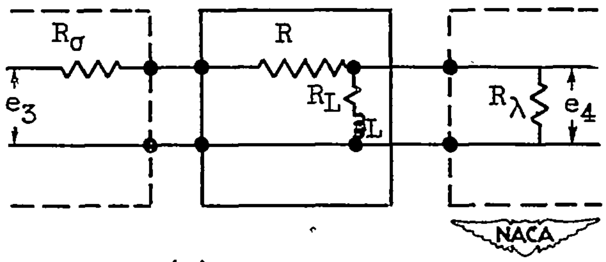


(a) RC compensator.



(b) RL compensator.

Figure 6. - RC and RL compensators connected to source and load.

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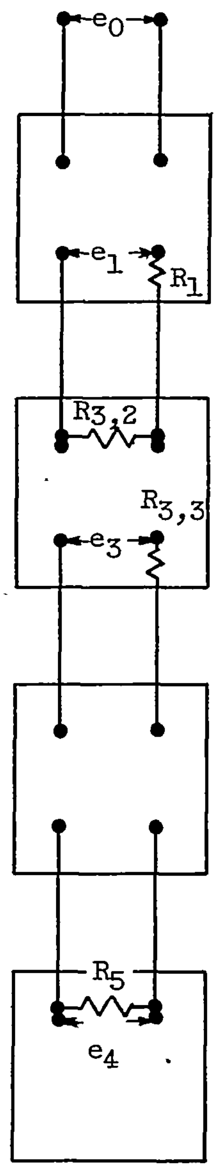
Network		Performance parameters	
Primary element		τ_1	time constant
		Q	emf per unit temperature change
Amplifier		μ_3	amplification factor
		$\omega_{3,a}$	low-frequency cut-off point
		$\omega_{3,b}$	high-frequency cut-off point
		$E_{2,N}$	equivalent rms input noise
Compensator		τ_4	time constant
		F_4	response improvement factor
		G_4	gain factor
Detector		δ_5	sensibility
		S_5	sensitivity

Figure 7. - RC or RL compensating system using amplifier.

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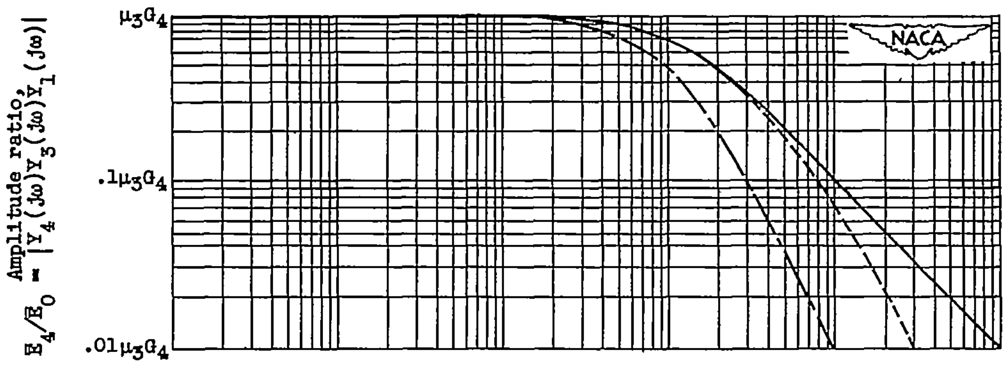
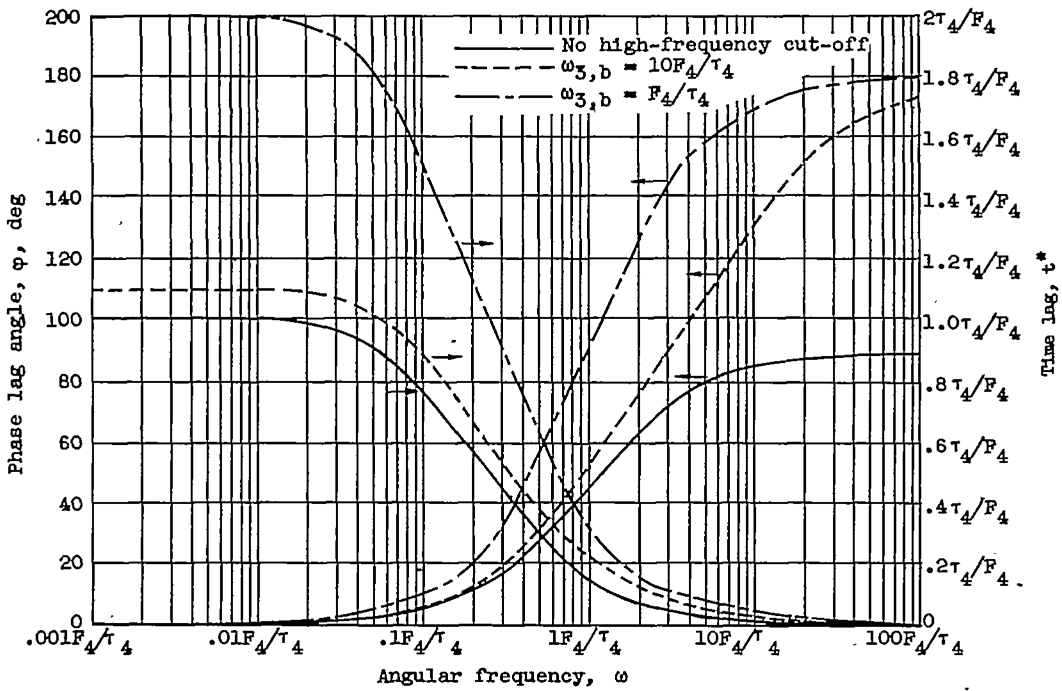
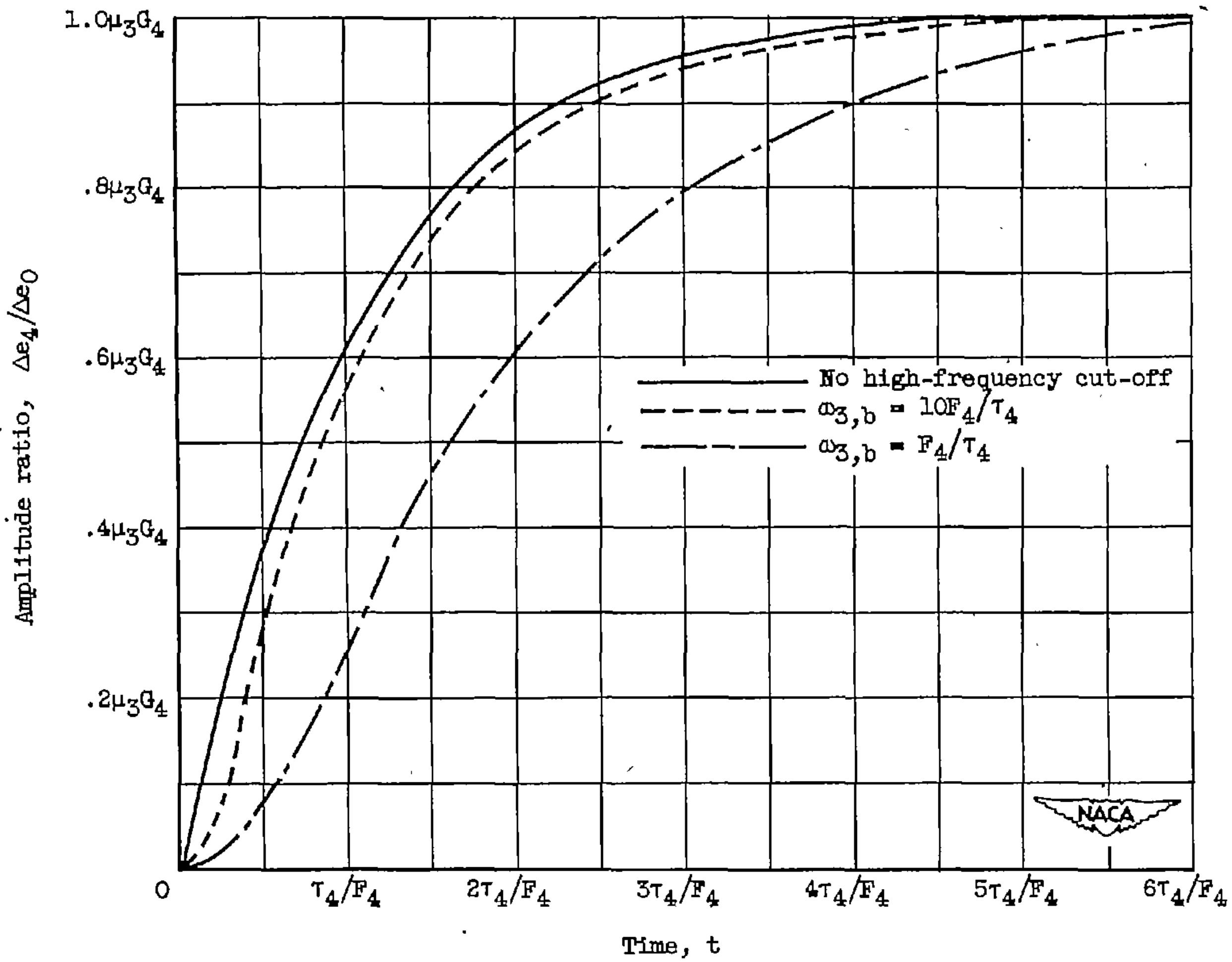


(a) Frequency-response relation; amplitude ratio.



(b) Frequency-response relation; phase and time lag.

Figure 8. - Response of RC or RL compensating system using amplifier with high-frequency cut-off. $\tau_4 = \tau_1$.



(c) Time history of response to step change.

Figure 8. - Concluded. Response of RC or RL compensating system using amplifier with high-frequency cut-off. $\tau_4 = \tau_1$.

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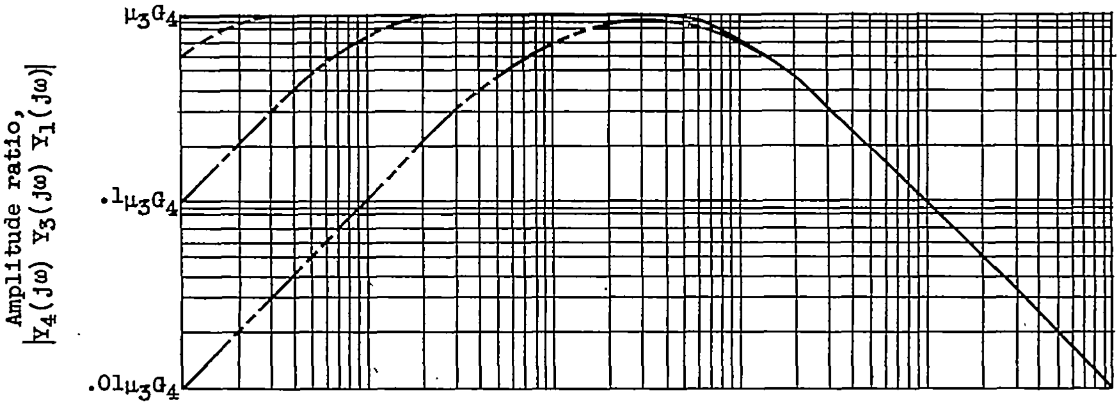
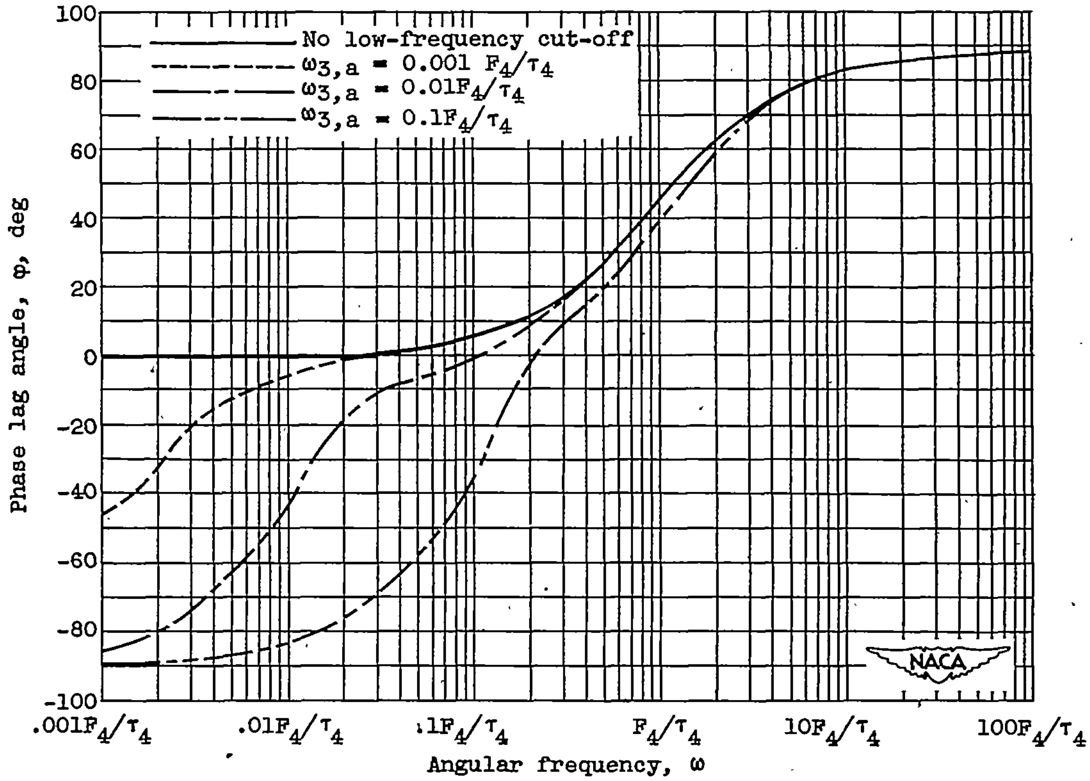


(a) Frequency-response relation; amplitude ratio.



(b) Frequency-response relations; phase lag angle.

Figure 9. - Response of RC or RL compensating system using amplifier with low-frequency cut-off. $\tau_4 = \tau_1$.

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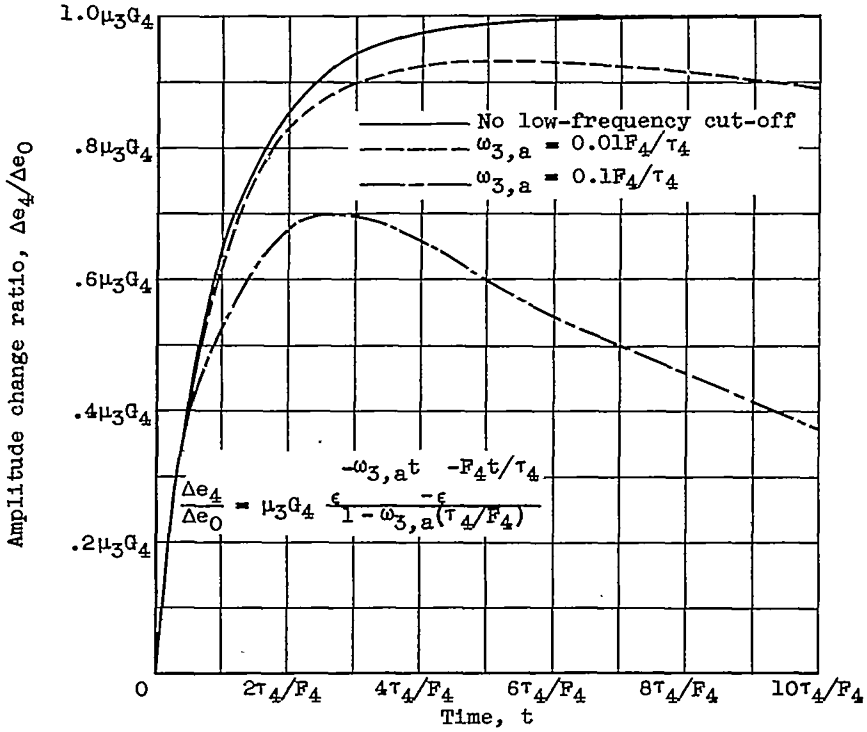
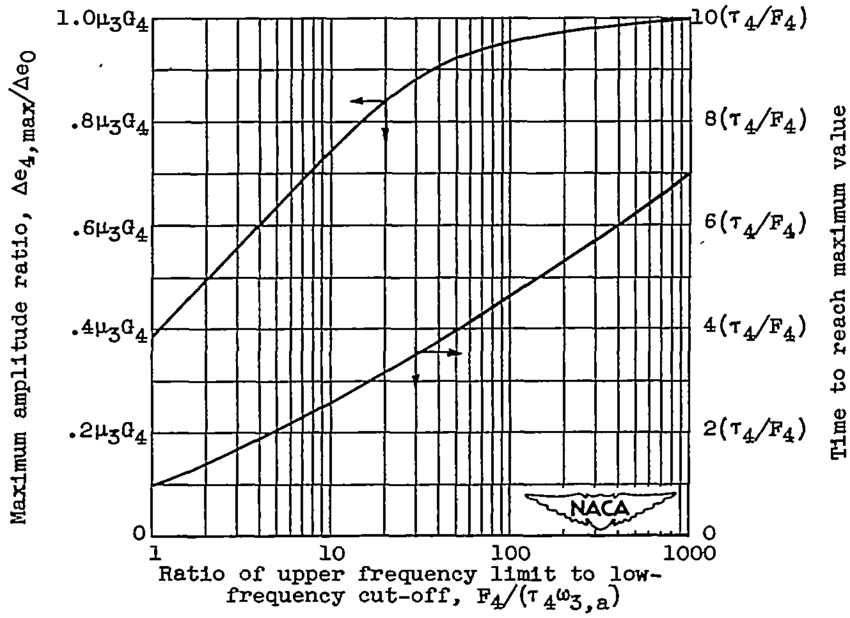


(c) Time history of response to step change.



(d) Maximum value reached and time to reach maximum value in response to step change.

Figure 9. - Concluded. Response of RC or RL compensating system using amplifier with low-frequency cut-off. $\tau_4 = \tau_1$.

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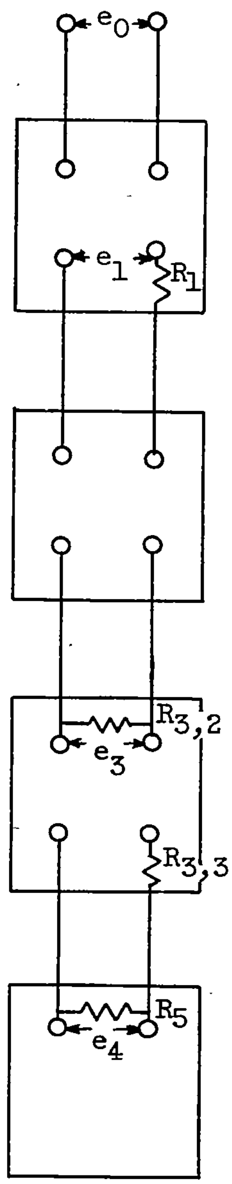

Network		Performance parameters
Primary element	τ_1	time constant
	Q	emf per unit temperature change
Compensator	τ_4	time constant
	F_4	response improvement factor
	G_4	gain factor
Amplifier	μ_3	amplification factor
	$\omega_{3,a}$	low-frequency cut-off point
	$\omega_{3,b}$	high-frequency cut-off point
	$E_{2,N}$	equivalent rms input noise
Detector	δ_5	sensibility
	S_5	sensitivity

Figure 10. - Compensating system with positions of amplifier and compensator interchanged.

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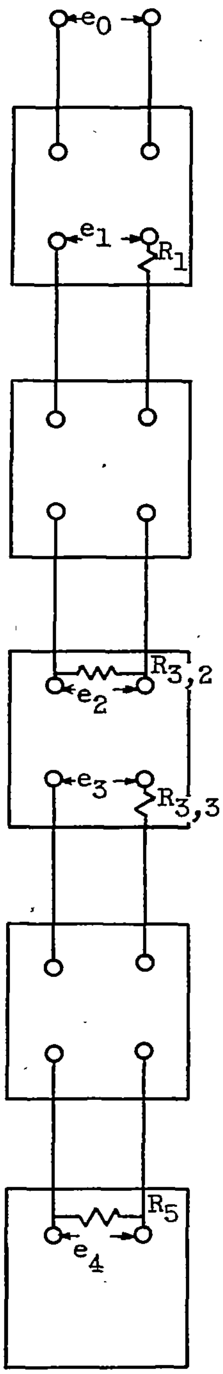

Network		Performance parameters
Primary element	τ_1	time constant
	Q	emf per unit temperature change
Transformer	N	turns ratio
	$\omega_{2,a}$	low-frequency cut-off point
	$\omega_{2,b}$	high-frequency cut-off point
Amplifier	μ_3	amplification factor
	$\omega_{3,a}$	low-frequency cut-off point
	$\omega_{3,b}$	high-frequency cut-off point
	$E_{2,N}$	equivalent rms input noise
Compensator	τ_4	time constant
	F_4	response improvement factor
	G_4	gain factor
Detector	δ_5	sensibility
	S_5	sensitivity

Figure 11. - Compensating system using transformer as noise-free preamplifier.

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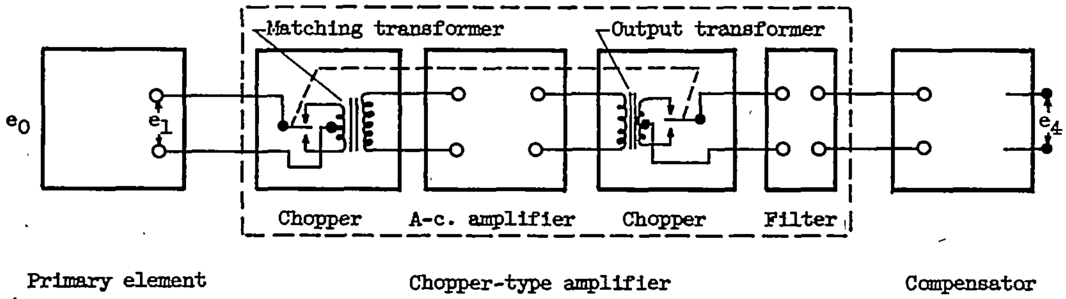
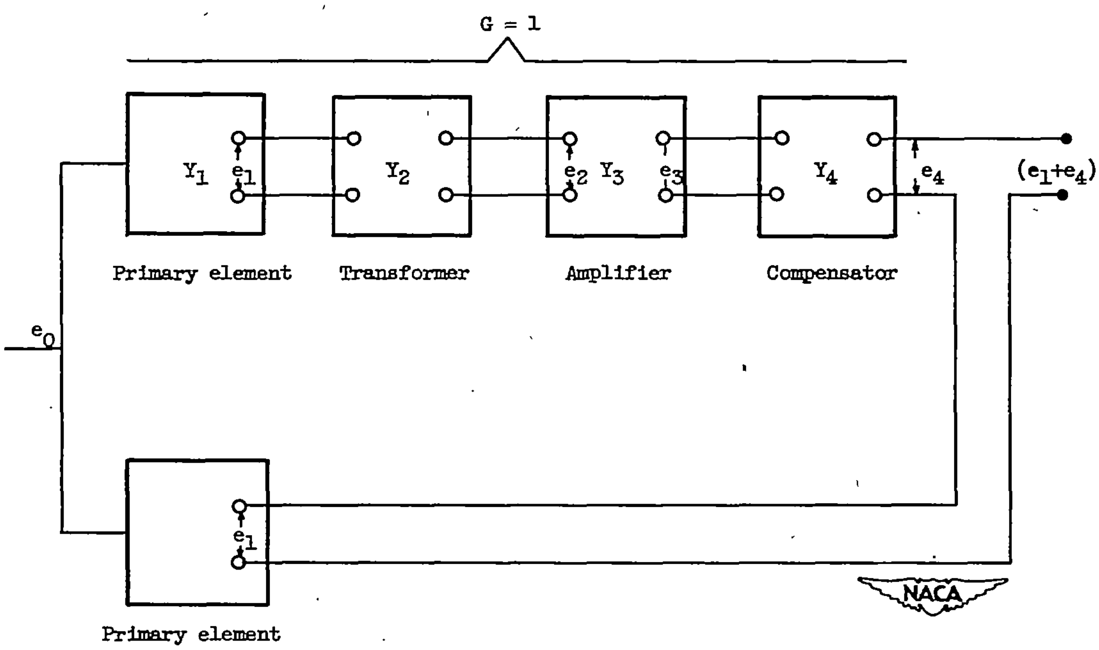


(a) Chopper-type amplifier.



(b) Addition of signal from second primary element.

Figure 12. - Techniques for maintaining d-c. response when transformer or a-c. amplifier is used.

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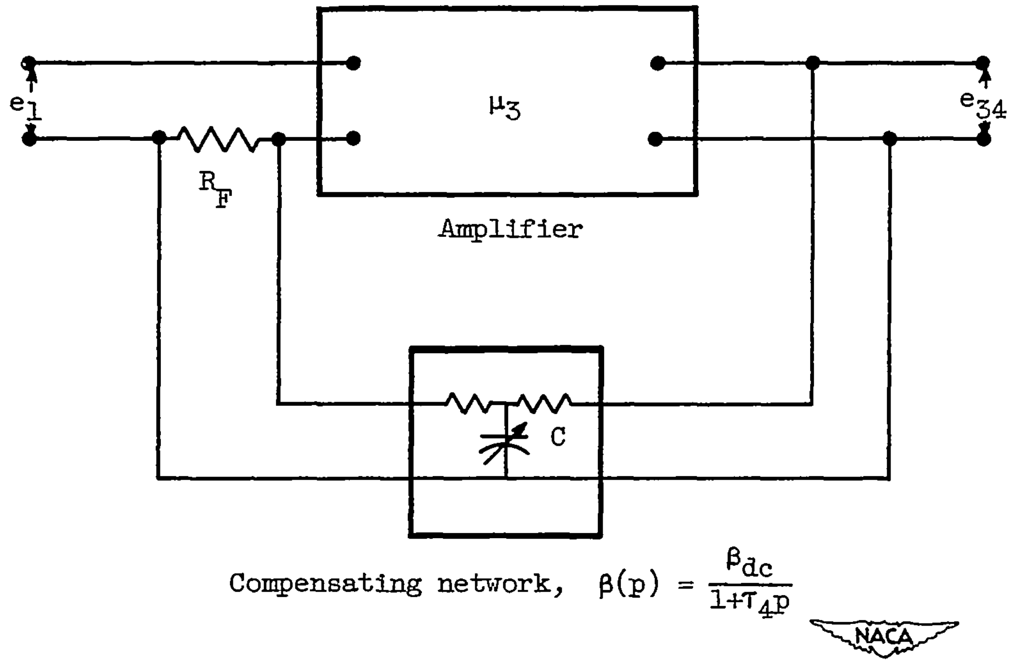

Figure 13. - Negative feedback amplifier with compensating network in feedback loop; β_{dc}, fraction of output voltage e_{34} appearing across R_F at zero frequency; τ_4, product of capacitance C and parallel resistance of output and input resistances of compensating filter.

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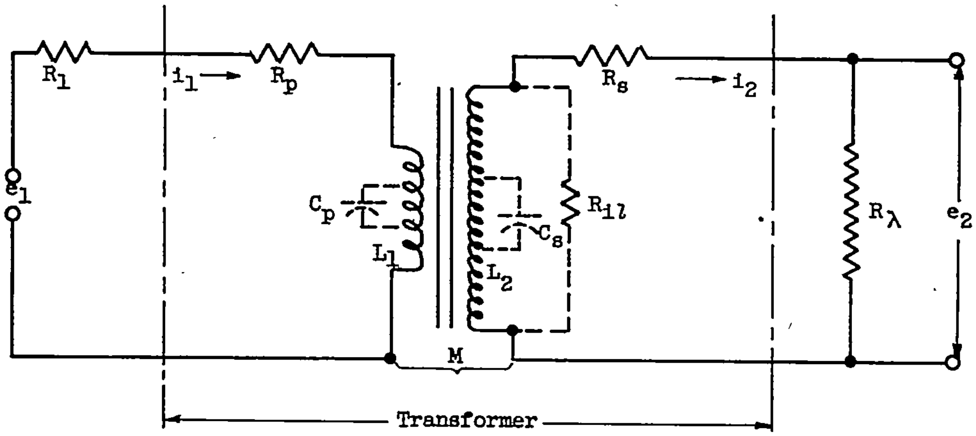
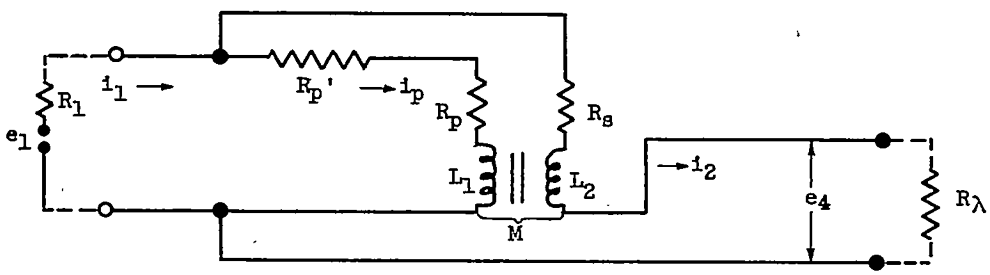
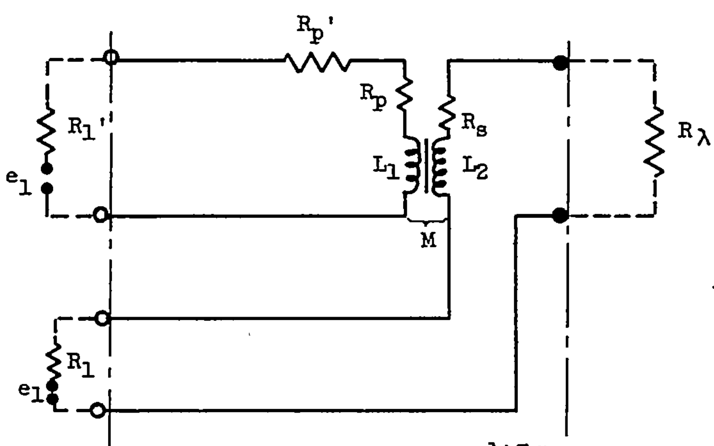


(a) Basic transformer circuit. Elements shown dashed may be neglected for most compensator applications. The iron losses are represented by $i_2^2 R_{il}$.



$$Y_{34} = G \frac{1+\tau_4 p}{1+(\tau_4/F)p}$$

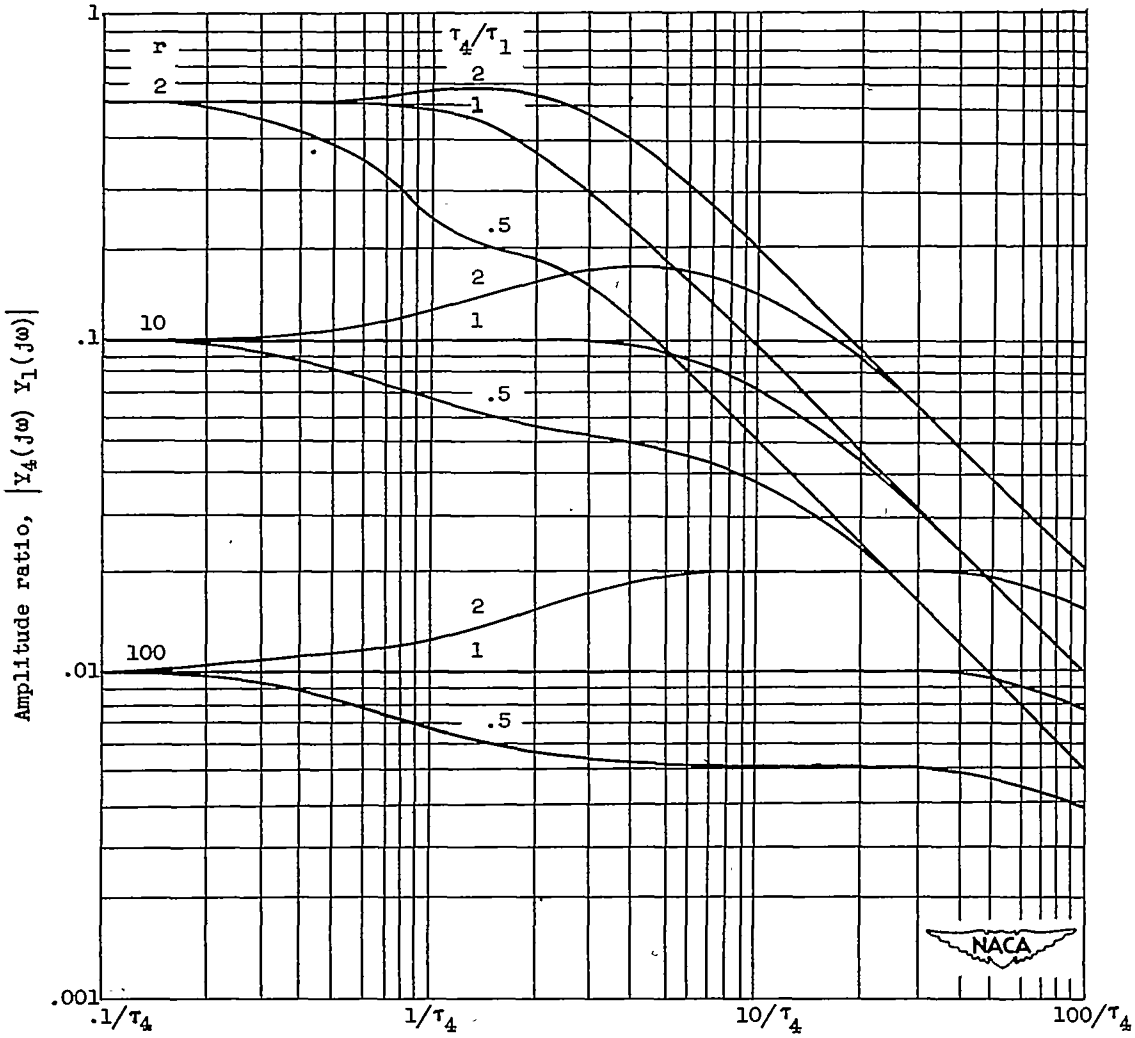
(b) One primary element.



$$Y_{34} = G \frac{1+\tau_4 p}{1+(\tau_4/F)p}$$

(c) Two primary elements.

Figure 14. - Transformer and its use as compensator.



(a) Frequency-response relation; amplitude ratio.

Figure 15. - Effect of mismatch between primary element and compensator time constants; F_4G_4 assumed equal to 1.

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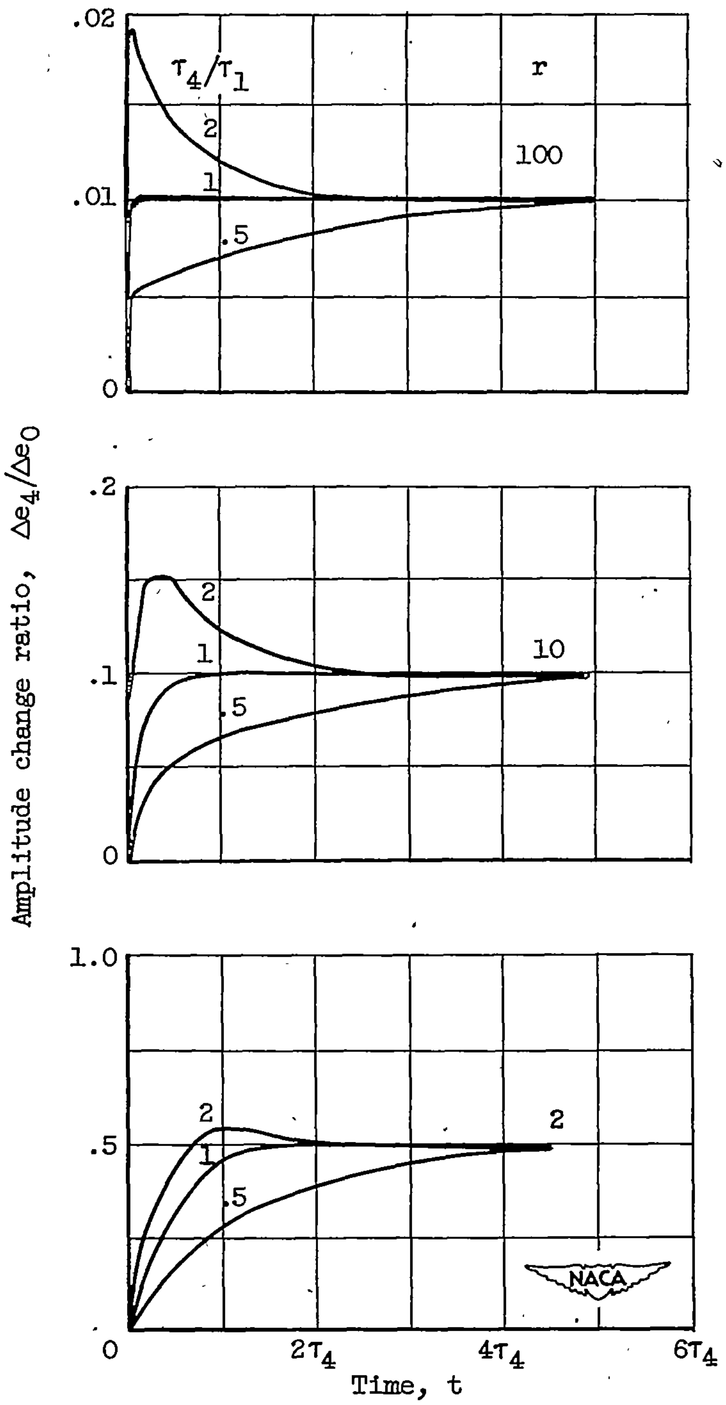
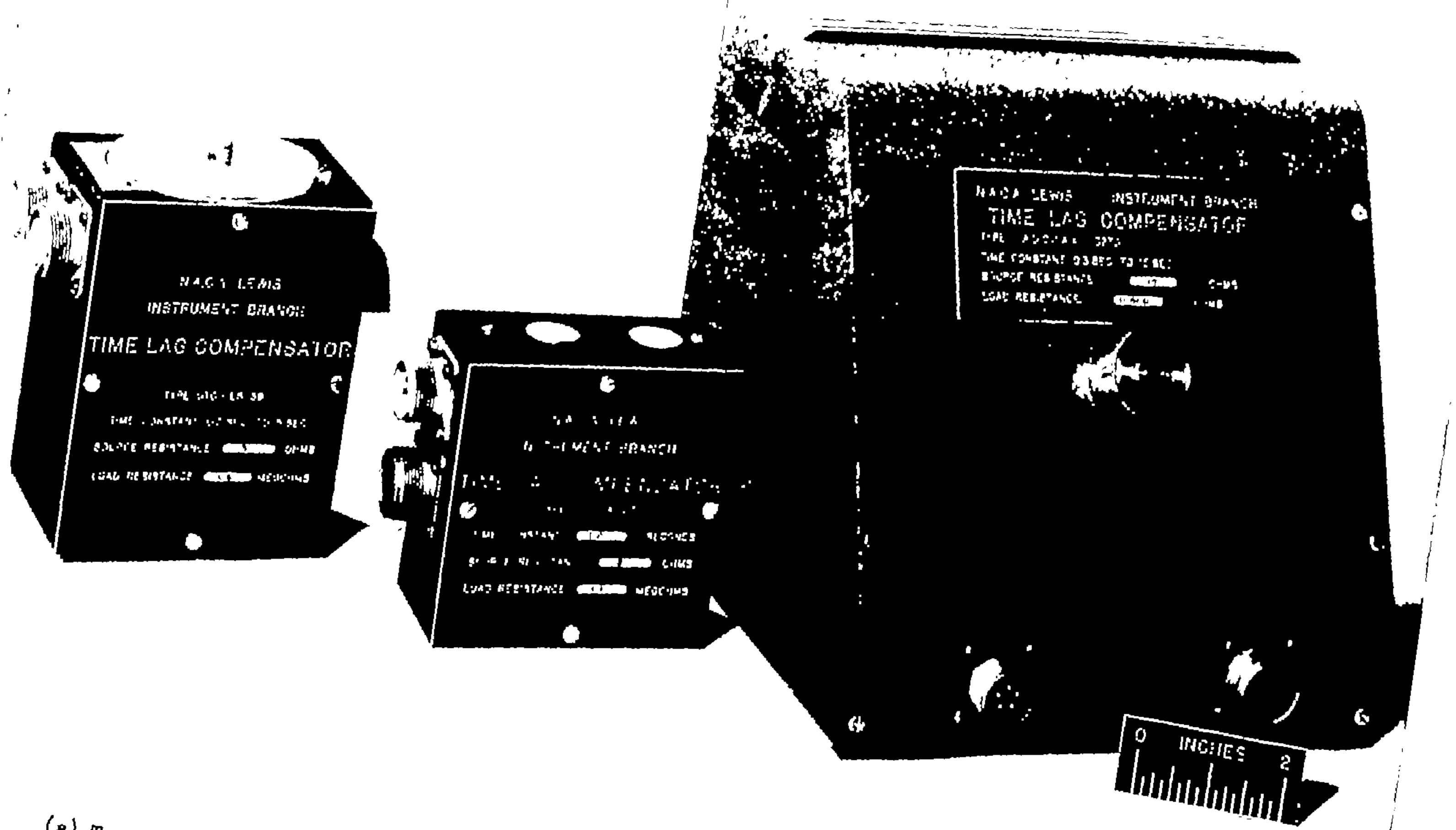


(b) Time history of response to step change.

Figure 15. - Concluded. Effect of mismatch between primary element and compensator time constants; F_4G_4 assumed equal to 1.



(a) Type UTC/LS 39.

(b) Type UTC/A 27.

(c) Type ADC/AX 3270.

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Figure 16. - Typical transformer-type compensators.

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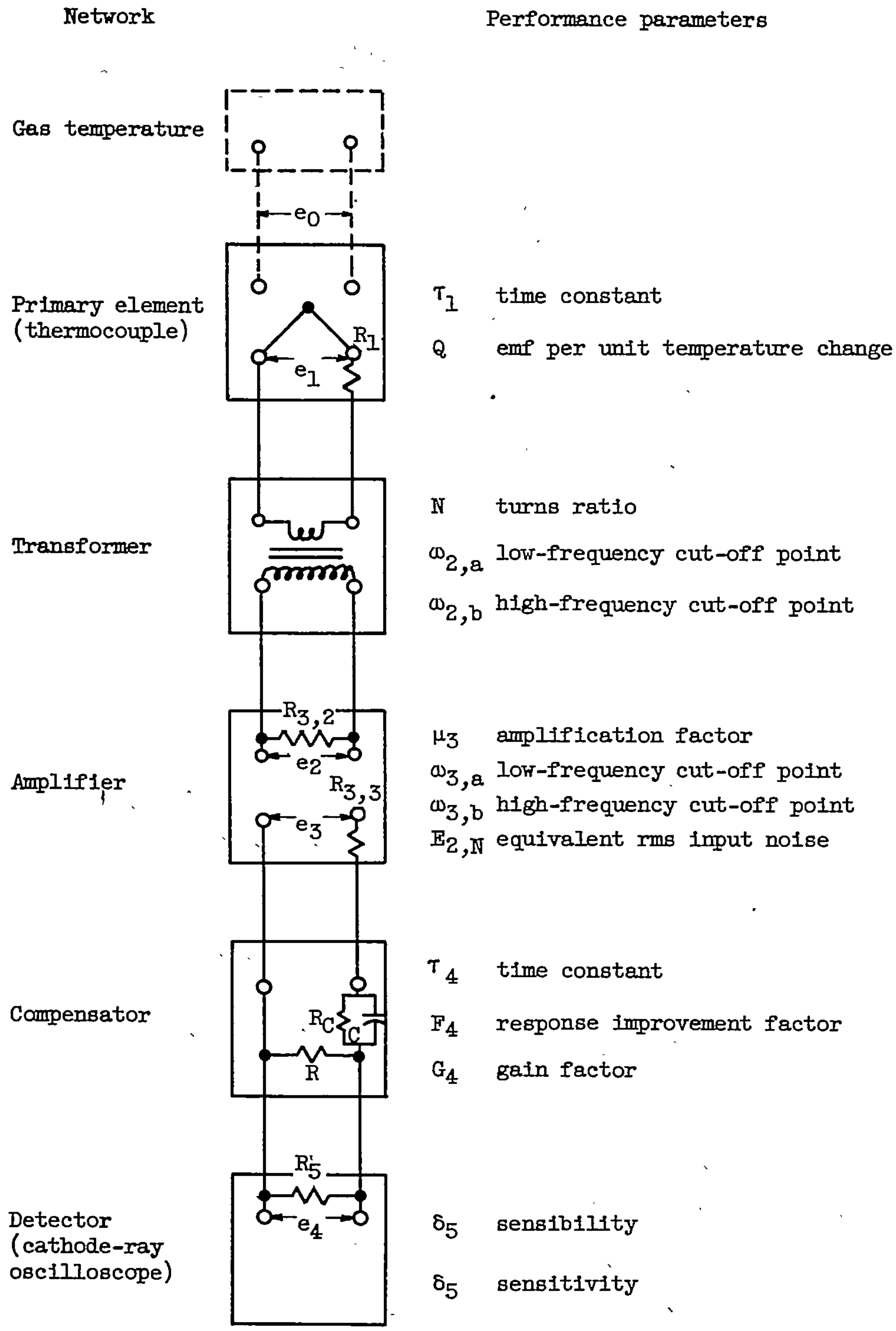
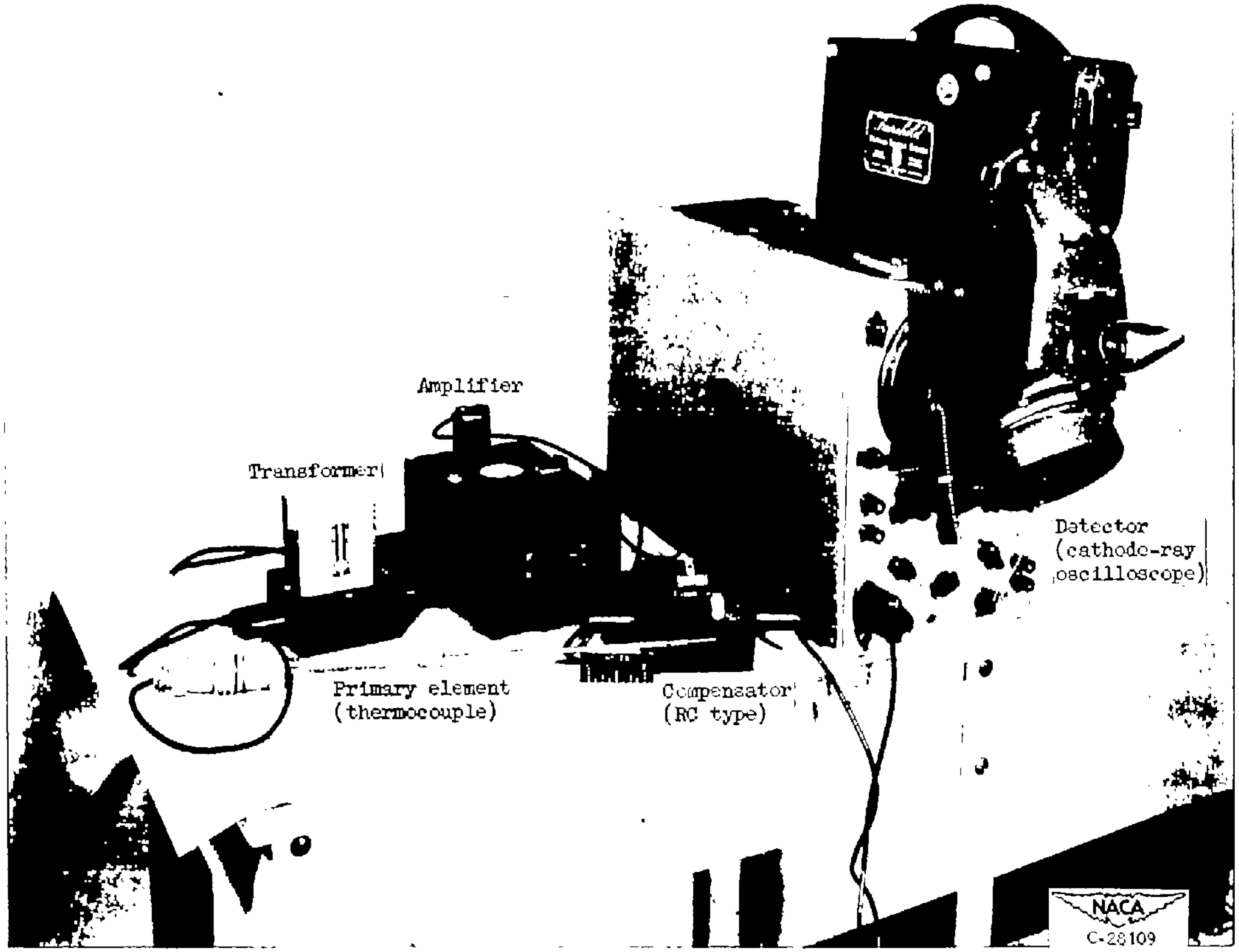


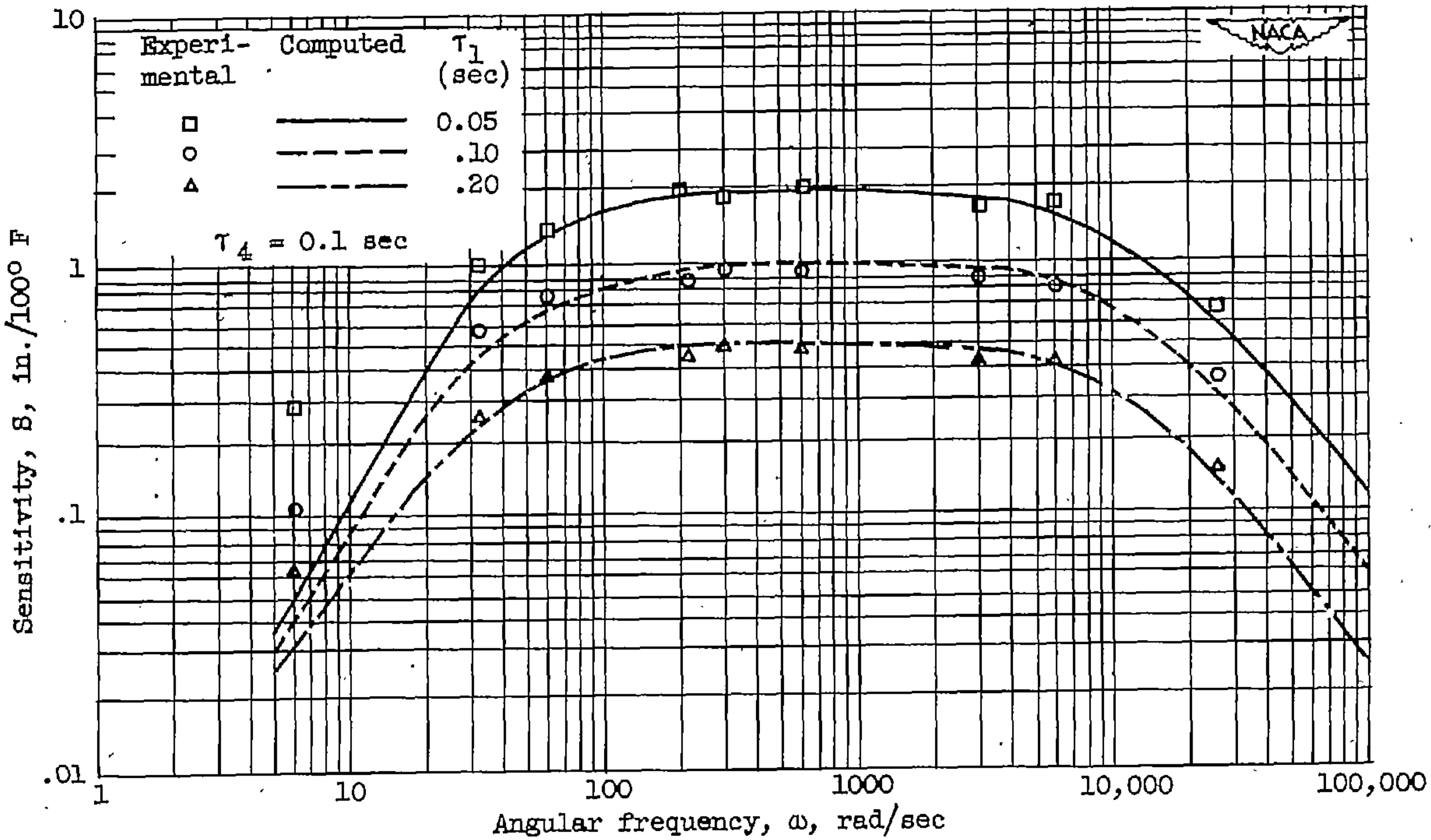
(a) Block diagram.

Figure 17. - Compensated system for measurement of alternating component of temperature.



(b) Representative arrangement of apparatus.

Figure 17. - Continued. Compensated system for measurement of alternating component of temperature.



(c) Frequency response relation; amplitude ratio.

Figure 17. - Continued. Compensated system for measurement of alternating component of temperature.

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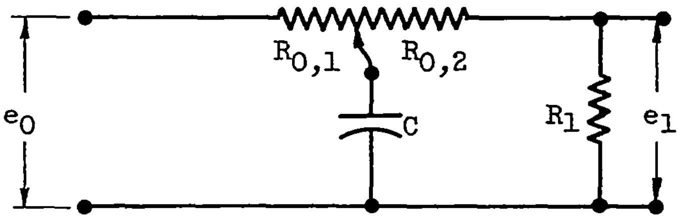


(d) Electrical analog of thermocouple with adjustable time constant.

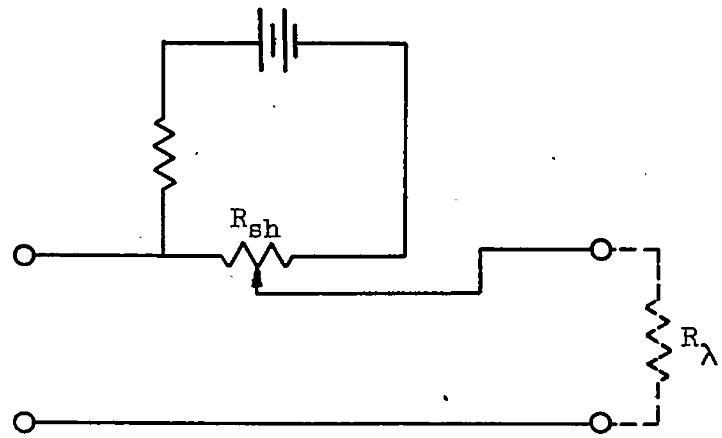
$$Y_1'(p) = \frac{e_1}{e_0} = G_1 \frac{1}{1+\tau_1 p}$$

where $\tau_1 = \dfrac{R_{0,1}(R_{0,2}+R_1)}{R_{0,1}+R_{0,2}+R_1} C$ and

$$G_1 = \frac{R_1}{R_{0,1}+R_{0,2}+R_1}$$

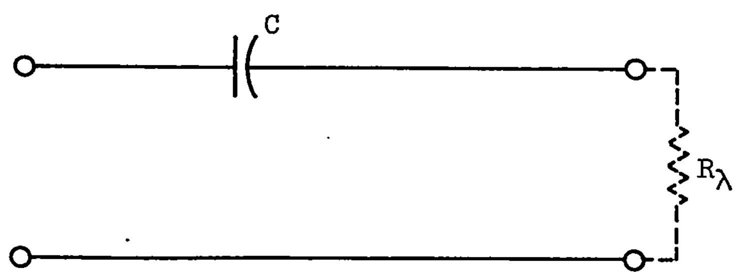
Figure 17. - Continued. Compensated system for measurement of alternating component of temperature.

2388



(e) Bucking circuit to reduce d-c. emf level.

$R_{sh} \ll R_\lambda$



(f) Capacitance coupling circuit to block d-c. signal.

$X_C \left(= \frac{1}{\omega C}\right)$ must be small compared with R_λ.

Figure 17. - Concluded. Compensated system for measurement of alternating component of temperature.

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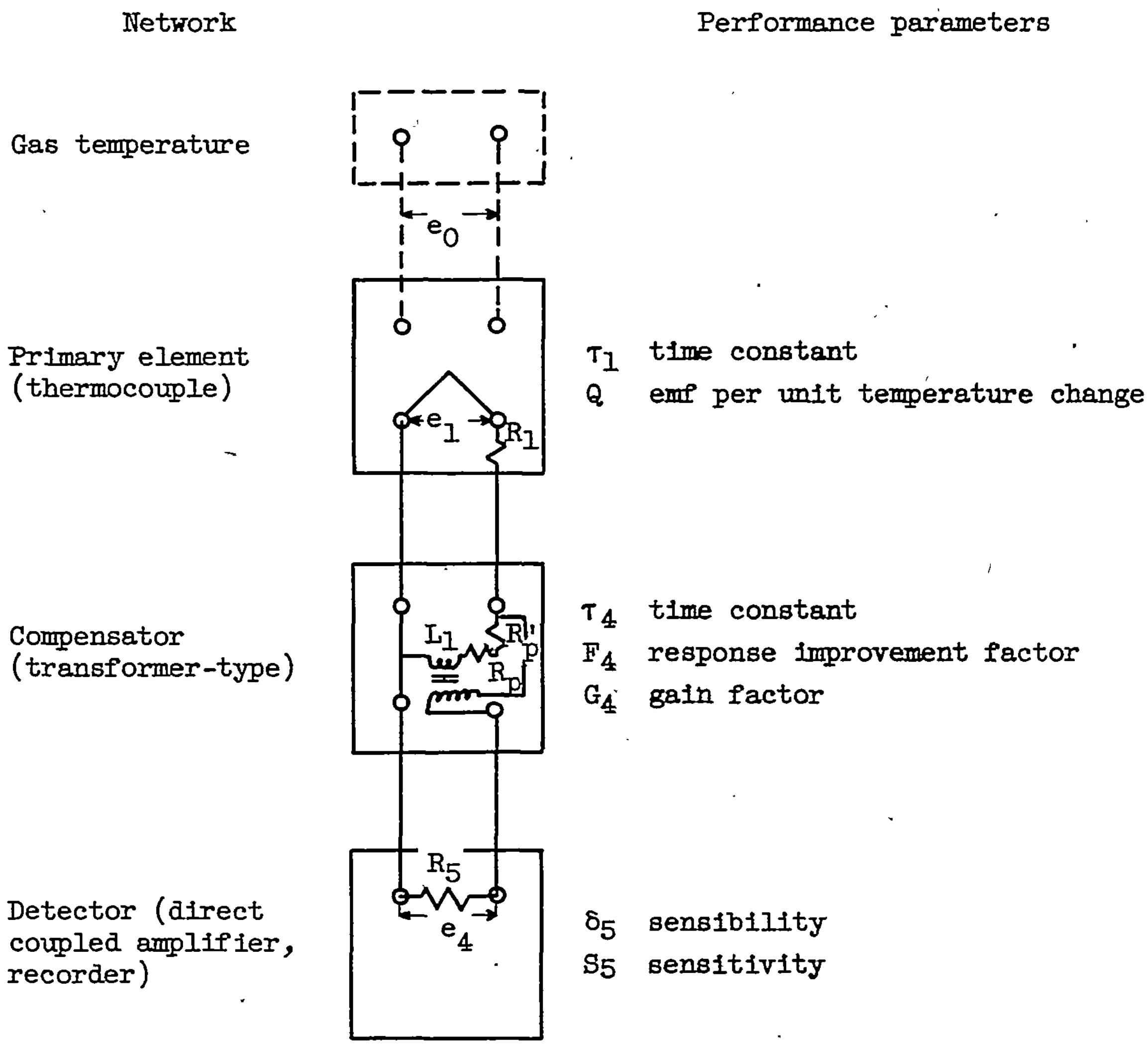
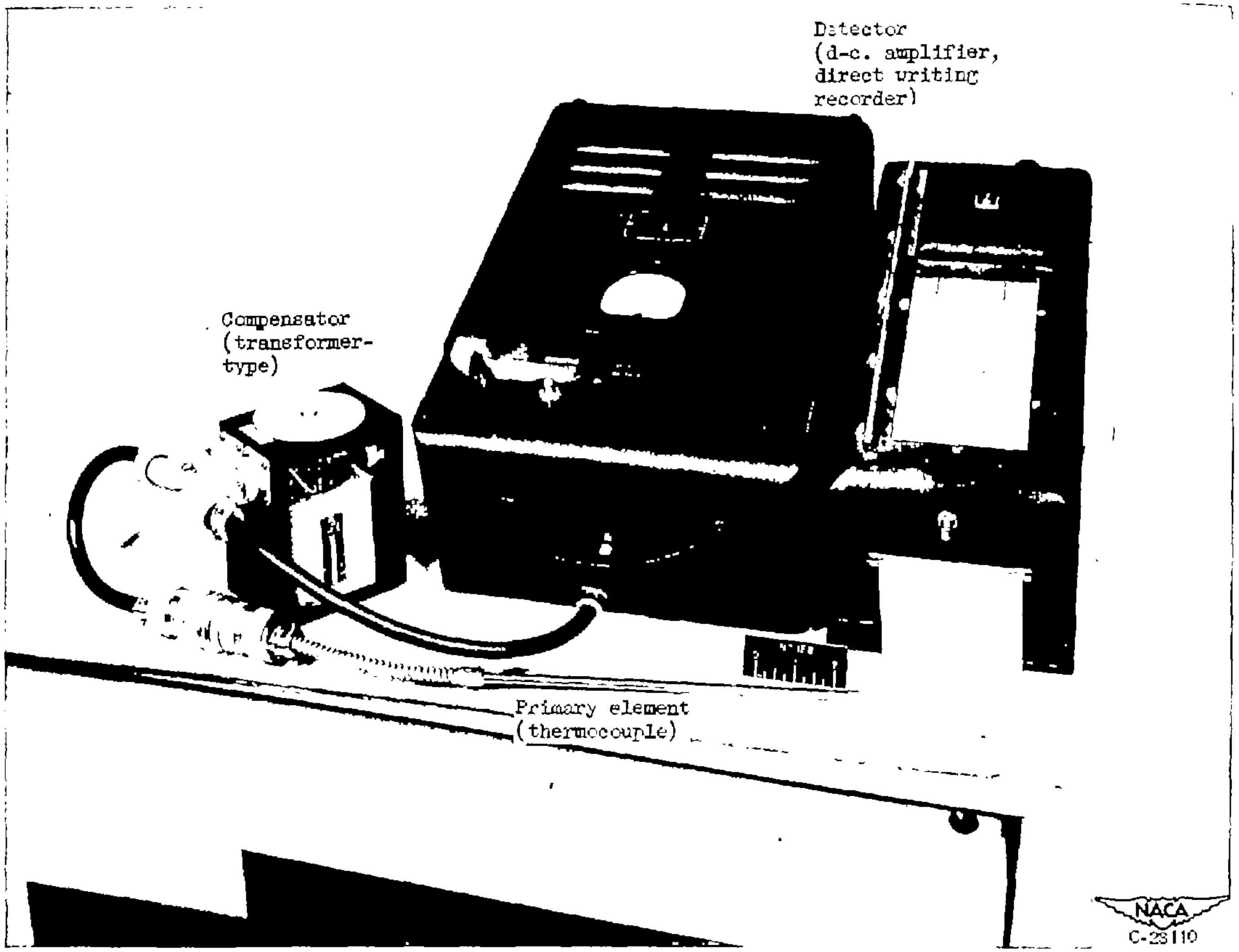


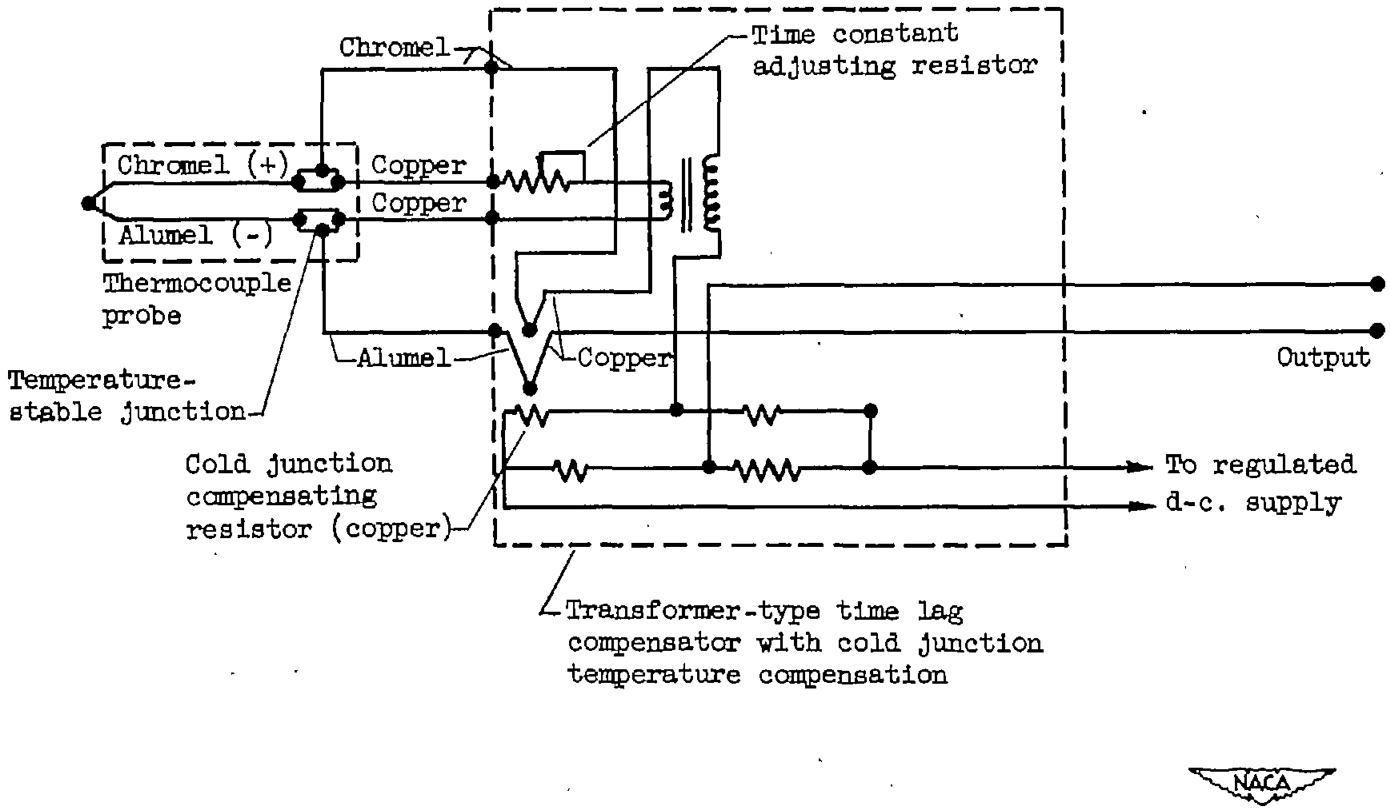
(a) Block diagram.

Figure 18. - Compensated system for measurement of average value as well as alternating component of temperature.



(b) Representative arrangement of apparatus.

Figure 18. - Continued. Compensated system for measurement of average value as well as alternating component of temperature.



(c) Reference-temperature compensation circuit for thermocouple.

Figure 18. - Continued. Compensated system for measurement of average value as well as alternating component of temperature.

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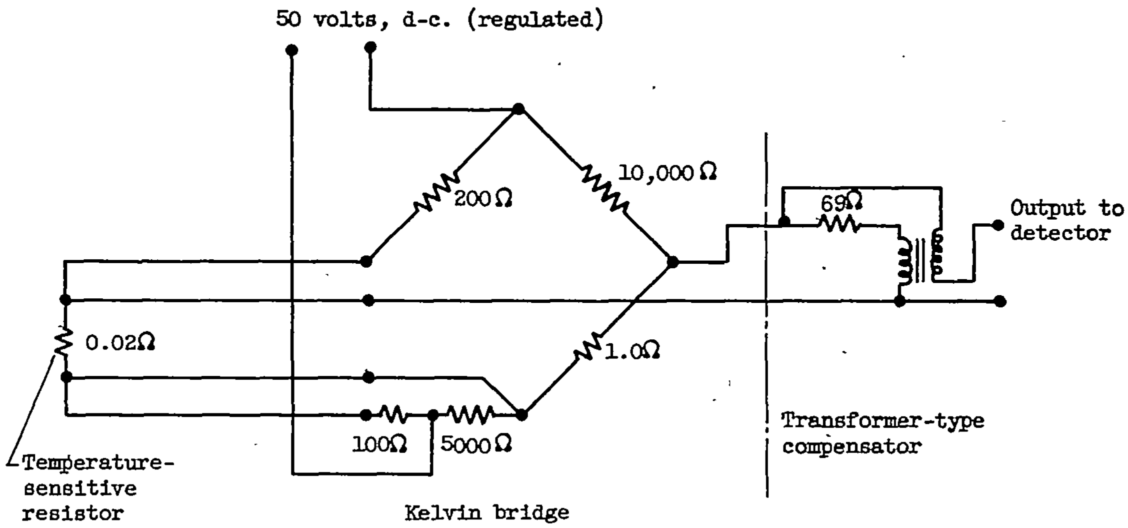
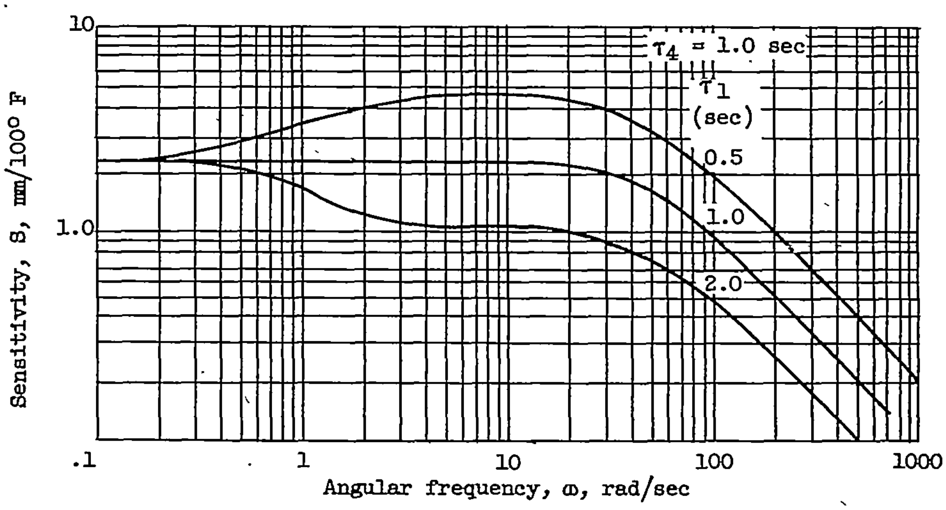


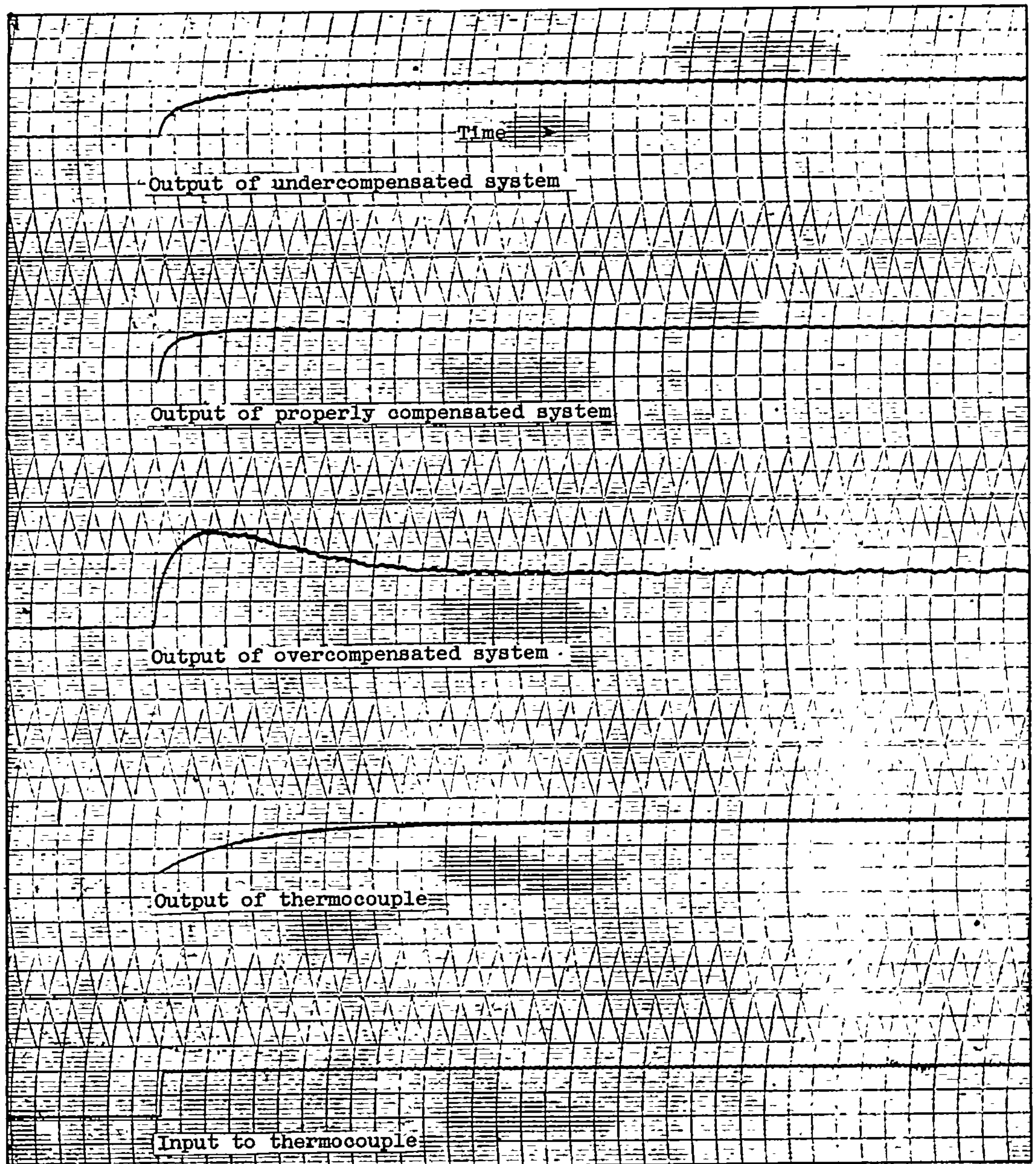
(d) Kelvin bridge circuit for resistance thermometer element.



(e) Frequency-response relation; amplitude ratio.

Figure 18. - Continued. Compensated system for measurement of average value as well as alternating component of temperature.

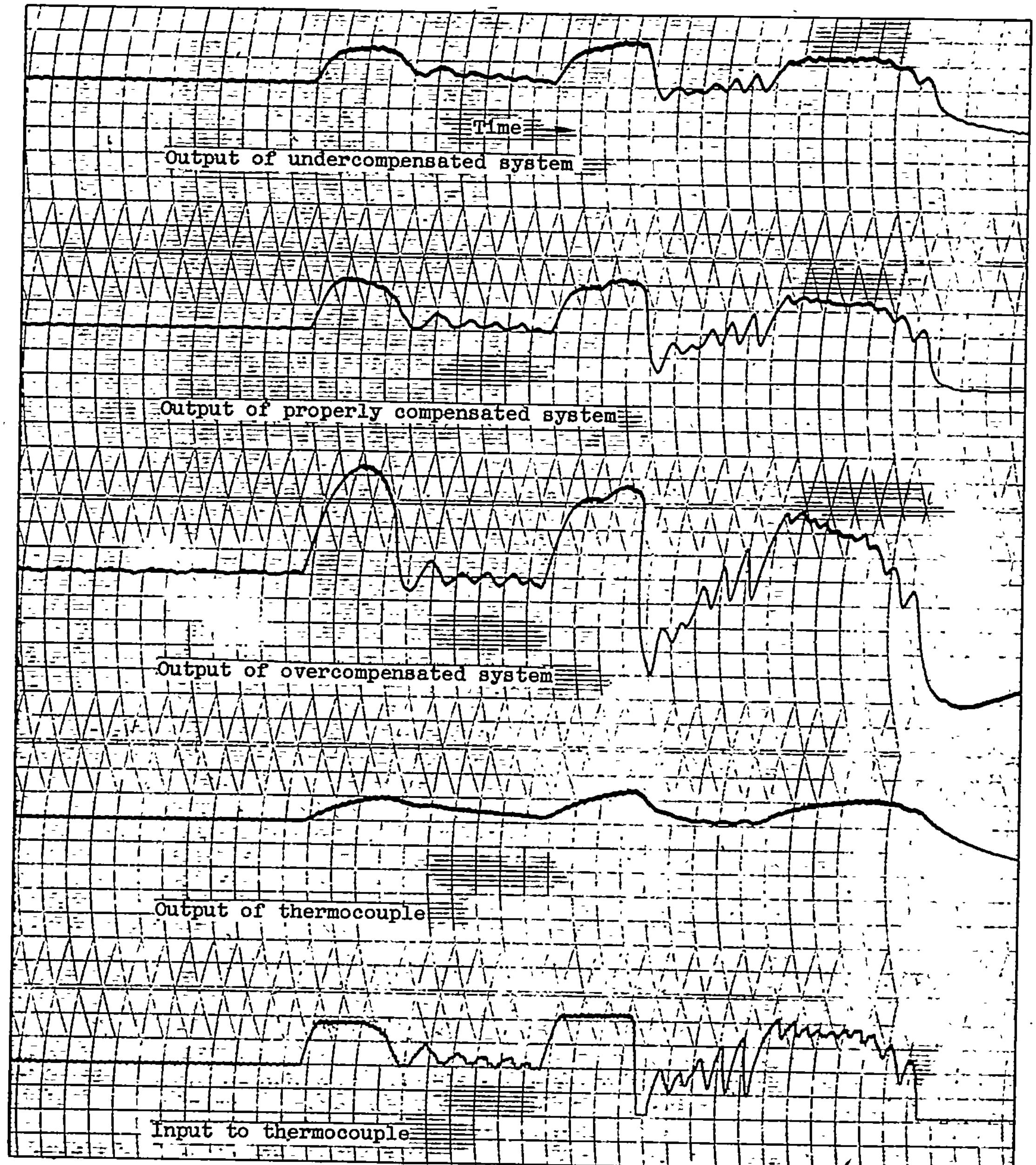
2388



(f) Time history of response to step change.

Figure 18. - Continued. Compensated system for measurement of average value as well as alternating component of temperature.

2388



(g) Time history of response to arbitrary input variation.

Figure 18. - Concluded. Compensated system for measurement of average value as well as alternating component of temperature.

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